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# 9.3: Separable Equations

Ex 1: $\frac{dy}{dx} = \frac{y}{x} \Rightarrow \frac{1}{y}\,dy = \frac{1}{x}\,dx$

$\Rightarrow \int \frac{dy}{y} = \int \frac{dx}{x}$

$\Rightarrow \ln|y| = \ln|x| + c.$

$\Rightarrow |y| = k|x|$ ~~**~~.

$\Rightarrow y = \pm kx$ ~~**~~. $\quad k = \pm e^c$

Ex 2: $(x^2+1)y' = xy$

Ex 3: $\frac{du}{dt} = 2 + 2u + t + tu.$

Ex 4: $\frac{dy}{dx} = y^2+1, \quad y(1) = 0.$

Ex 5: $\frac{dp}{dt} = \sqrt{pt}, \quad p(1) = 2$

Ex 6: $xy' + y = x^2, \quad y(1) = -1$

} Initial value problems.

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## Mixing Problem

Ex7: A small country has $10 bil in paper currency in circ, and each day $50 mil comes into banks. The government decides to replace old bills w/new when the old comes into banks. How long until the new bills account for 90% of the currency in circ?

Let $X = x(t)$ denote the amount of new currency in circ $t$ days after the release of the new bills.

$x(0) = 0$.

solve $X' = \frac{10\,bil - X}{10\,bil} \cdot 50\,mil$

$$\Rightarrow \frac{dX}{10bil - X} = \frac{50mil}{10bil}\, dt$$

$$\Rightarrow \int \frac{dX}{10bil - X} = \int \frac{50mil}{10bil}\, dt$$

$$\Rightarrow -\ln|10bil - x| = \frac{50mil}{10bil} t + C$$

$$\Rightarrow |10bil - X| = k e^{-\frac{5}{1000}t}$$

$$\Rightarrow X = 10bil - k e^{-5/1000\, t} \qquad X(0) = 0 \text{ gives } k = 10bil$$

$$\Rightarrow X = 10bil\left(1 - e^{-5/1000\, t}\right)$$

$$\Rightarrow 0.9 = 1 - e^{-t/200}$$

$$\Rightarrow 0.1 = e^{-t/200}$$

$$\Rightarrow -200 \ln(0.1) = t \approx 460.5 \text{ days.}$$