9.1 HW
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[2] verify $y = \sin x \cos x - \cos x$ is a solution to IVP

$$y' + y \tan x = \cos^2 x \; ; \quad y(0) = -1$$

<u>solution</u>

$$y' = \cos^2 x + - \sin^2 x + \sin x$$

$$y' + y \tan x = \cos^2 x - \sin^2 x + \sin x + (\sin x \cos x - \cos x)\tan x$$

$$= \cos^2 x - \cancel{\sin^2 x} + \cancel{\sin x} + \cancel{\sin^2 x} - \cancel{\sin x}$$

$$= \cos^2 x$$

AND $y(0) = 0 \cdot 1 - 1 = -1$ ✓

[3] (a) when does $y = e^{rx}$ satisfy $2y'' + y' - y = 0$

$$2r^2 e^{rx} + r e^{rx} - e^{rx} = 0$$

$$\Rightarrow 2r^2 + r - 1 = 0$$

$$\Rightarrow (2r - 1)(r + 1) = 0$$

$$\Rightarrow r = \tfrac{1}{2} \text{ or } r = -1$$

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(b) verify $y = ae^{\frac{1}{2}x} + be^{-x}$ is a solution

$$\Rightarrow y' = \frac{1}{2}ae^{\frac{1}{2}x} - be^{-x}$$

$$\Rightarrow y'' = \frac{1}{4}ae^{\frac{1}{2}x} + be^{-x}$$

$$2y'' + y' - y = 2\left(\frac{1}{4}ae^{\frac{1}{2}x} + be^{-x}\right) + \left(\frac{1}{2}ae^{\frac{1}{2}x} - be^{-x}\right) - \left(ae^{\frac{1}{2}x} + be^{-x}\right)$$

$$= \cancel{\frac{1}{2}ae^{\frac{1}{2}x}} + \cancel{2be^{-x}} + \cancel{\frac{1}{2}ae^{\frac{1}{2}x}} - be^{-x} - \cancel{ae^{\frac{1}{2}x}} - \cancel{be^{-x}}$$

$$= 0.$$

$\boxed{5}$ which are solutions to $y'' + y = \sin x$

a) $y = \sin x$: $-\sin x + \sin x = \sin x$ ✓

b) $y = \cos x$; $-\cos x + \cos x = \sin x$ ✗

c) $y = \frac{1}{2}x\sin x$; $\frac{1}{2}\sin x + \frac{1}{2}x\cos x + \frac{1}{2}x\sin x$

$$= \frac{1}{2}\left(\sin x + x(\cos x + \sin x)\right)$$

$$\neq \sin x \quad ✗$$

d) $y = -\frac{1}{2}x\cos x$; $-\frac{1}{2}\cos x + \frac{1}{2}x\sin x - \frac{1}{2}x\cos x$

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c) $y = \frac{1}{2} x \sin x$

$\Rightarrow y' = \frac{1}{2} \sin x + \frac{1}{2} x \cos x$

$\Rightarrow y'' = \frac{1}{2} \cos x + \frac{1}{2} \cos x - \frac{1}{2} x \sin x$

$= \cos x - \frac{1}{2} x \sin x$.

so $y'' + y = \cos x - \frac{1}{2} x \sin x + \frac{1}{2} x \sin x$

$= \cos x$

d) $y = -\frac{1}{2} x \cos x$

$\Rightarrow y' = -\frac{1}{2} \cos x + \frac{1}{2} x \sin x$

$\Rightarrow y'' = +\frac{1}{2} \sin x + \frac{1}{2} \cancel{x} \sin x + \frac{1}{2} x \cos x$

so $y'' + y = + \sin x + \frac{1}{2} x \cos x \mp \frac{1}{2} x \cos x$

$= + \sin x$

only (d) is a solution.

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$\boxed{6}$ a) Show that $y = \frac{\ln x + c}{x}$ is a solution to $x^2 y' + xy = 1$

$$y' = \frac{\frac{1}{x} \cdot x - 1(\ln x + c)}{x^2}$$

$$= \frac{1 - \ln x - c}{x^2}$$

so $x^2 y' + xy = \cancel{x^2}\left(\frac{1 - \ln x - c}{\cancel{x^2}}\right) + \cancel{x}\left(\frac{\ln x + c}{\cancel{x}}\right)$

$$= 1 - \cancel{\ln x} - \cancel{c} + \cancel{\ln x} + \cancel{c}$$

$$= 1 \checkmark$$

b)

c) $y(1) = 2$: $2 = \frac{\ln(1) + c}{1} \Rightarrow c = 2$.

d) $y(2) = 1$: $1 = \frac{\ln(2) + c}{2} \Rightarrow c = \cancel{\frac{2}{\ln 2}}$ $2 - \ln 2$

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[8] $y' = xy^3$ when x is near zero
when x is large.

a) when x is near zero, the graph of y will be fairly flat ($y' \approx 0$). when x is large, ~~so is y' so so y will be increasing quickly (or decreasing as the case may be)~~. we don't know (need more info).

b) verify $y = (c - x^2)^{-1/2}$ is a solution.

$$y' = -\frac{1}{2}(c - x^2)^{-3/2} \cdot (-2x)$$

$$= \frac{x}{(c-x^2)^{3/2}}$$

$$\text{so } xy^3 = x \cdot \frac{1}{[(c-x^2)^{1/2}]^3}$$

$$= \frac{x}{(c-x^2)^{3/2}}$$

c) yes

d) $y(0) = 2$; $2 = (c - 0^2)^{-1/2} \Rightarrow c = 1/4$

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[9] $\frac{dP}{dt} = 1.2P\left(1 - \frac{P}{4200}\right)$

a) P increases on

$0 \leq P < 4200$

b) P decreases on $P > 4200$

c) equilibrium solutions

$P = 0$ or $P = 4200$

(where $P' = 0$).

[11] Why graphs can't be solutions to $y' = e^t(y-1)^2$

a) If $(y-1)^2 > 0$

then $y' > 0$

so the graph

should be <u>increasing</u>.

(or @ least non-decreasing).

b) @ $y = 1$

$\frac{dy}{dt} = 0$ ...

but the graph

isn't flat.

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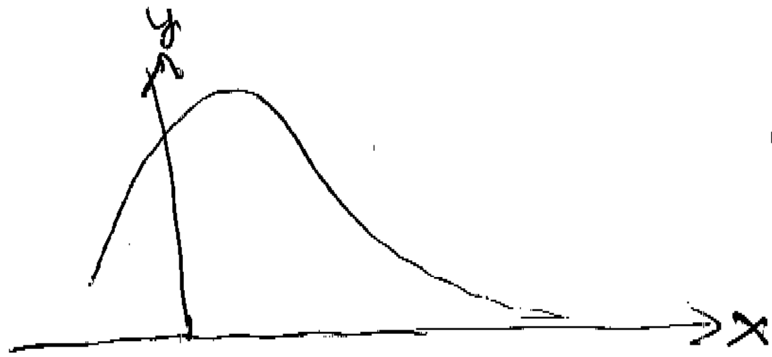

A) $y' = 1 + xy$. No

if x & y positive (1st Q), the fct. would have to be increasing.

B) $y' = -2xy$. No

same as (A), but just look @ the increasing segment in Q 1.

c) $y' = 1 - 2xy$. Yes

Allows for pos, neg, & zero slopes in Q 1.

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$\boxed{T4}$ initial temp of coffee : 95°c
room temp is 20°c.

a) cools most quickly @ t=0
when the temp. difference (75°c)
is greatest. As time goes by, the
rate of cooling will decrease to zero.

b) $\frac{dT}{dt} = k(T - T_s)$

$\Rightarrow \frac{dT}{dt} = k(T - 20)$

where $T(0) = 95°c$.

And I think this is a fantastic model.

c)