8.3
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The weighted average.

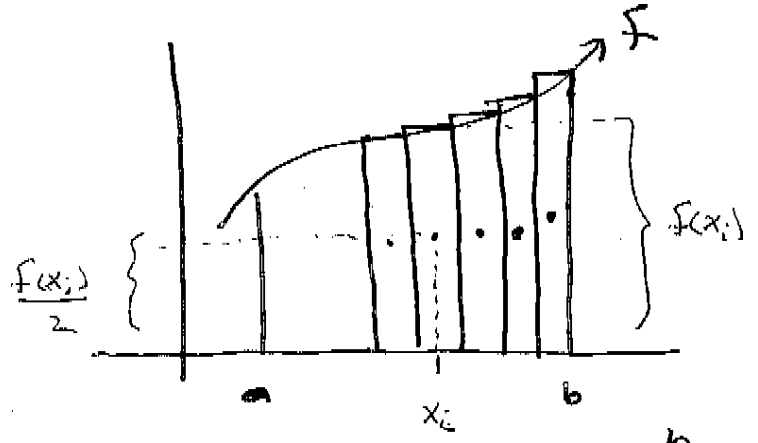
Dusty's grade "C" giving into the final to get a 4.0.

HW
Test
Final.

Grade if

HW	10	90
Q	10	85
T	45	80
A	5	95
F	30	82

Find the center of mass of a plate w/ uniform density ρ per unit area.



$$M_y = \rho \int_a^b x f(x)\,dx$$

$$M_x = \rho \int_a^b \frac{1}{2}[f(x)]^2\,dx$$

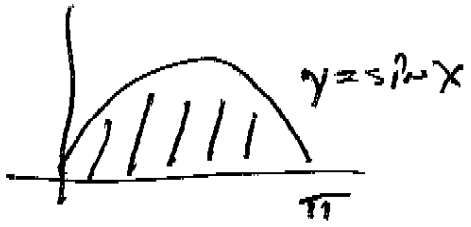
$$\bar{x} = \frac{M_y}{\rho A} \leftarrow \text{mass}$$

$$\bar{y} = \frac{M_x}{\rho A} \leftarrow \text{mass}$$

Derive $\bar{x} = \frac{1}{A}\int_a^b x f(x)\,dx$

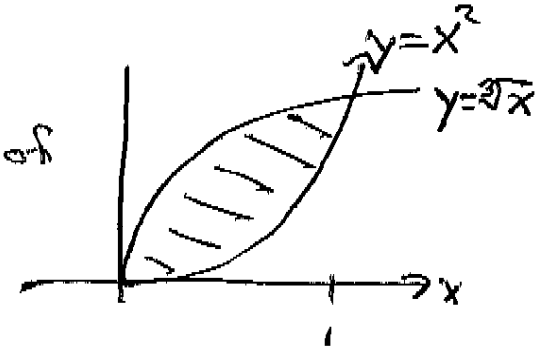
$\bar{y} = \frac{1}{A}\int_a^b \frac{1}{2}[f(x)]^2\,dx$

Ex1: Find the centroid of



The moments,

Ex2: Find the centroid of



8.3
2/7

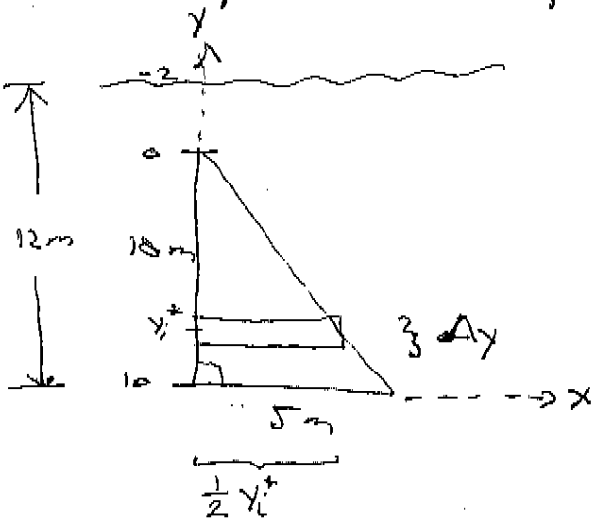
Hydrostatic Pressure

Pressure increases linearly due to the mass of the water above.

The important fact is that at any point in a liquid the pressure is the same in all directions.

So $P = \rho g d$ (d = depth).

Ex 1: Find the hydrostatic force on the vertical plate submerged in the water



$A_i = \frac{1}{2} y_i^* \Delta y$

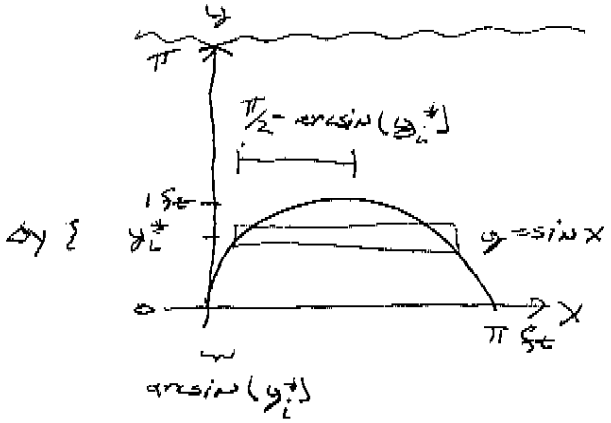
$P_i = 1000(10)(2 + y_i^*)$

$F_i = 5000\, y_i^*(2 + y_i^*)\Delta y = A_i \cdot P_i$

$$F = \lim_{N\to\infty} \sum_{i=1}^{N} 5000\, y_i^*(2+y_i^*)\Delta y$$

$$= 5000 \int_0^{10} y(2+y)\,dy.$$

Ex 2: Find the hydrostatic force on the plate.



$A_i^* = 2\left(\frac{\pi}{2} - \arcsin(y_i^*)\right)\Delta y$

$P_i = 62.5(\pi - y_i^*)$

$$F = \int_0^1 62.5(\pi - y_i^*)\, 2\left(\frac{\pi}{2} - \arcsin y\right) dy.$$

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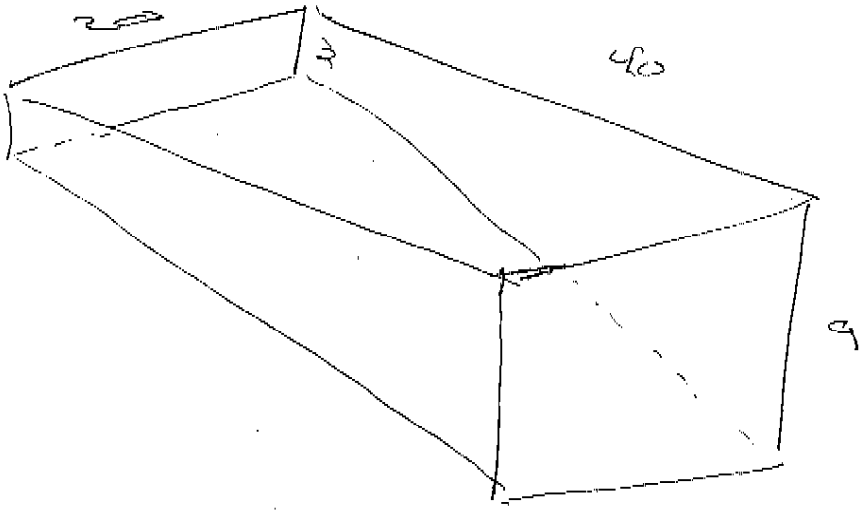
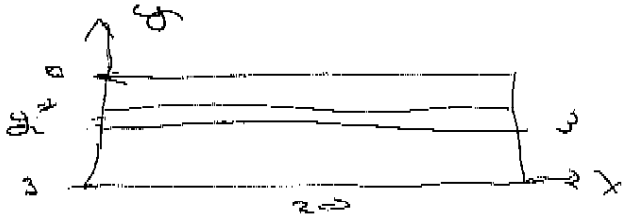
15. Swimming Pool.
Find the pressure on all ends.





a) shallow end

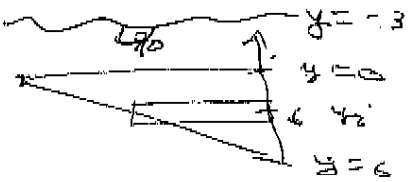
$A_i^* = 20\,\Delta y$.

$P_i^* = 62.5\, y_i^*$

$\int_0^3 20(62.5)\, y\, dy = 20(62.5)\frac{9}{2} = 5625$ lbs.

b) $20(62.5)\,9/2 = 50625$ lbs (deep end)

c) side $40(62.5)\,9/2$ = 11250



$A_i = (40 - \frac{40}{6} y_i)\,\Delta y$

$P_i = 62.5(3 + y_i)$

$\int_0^6 62.5(3+y)(40 - \frac{40}{6} y)\, dy = 37500 = 48750$ lbs.

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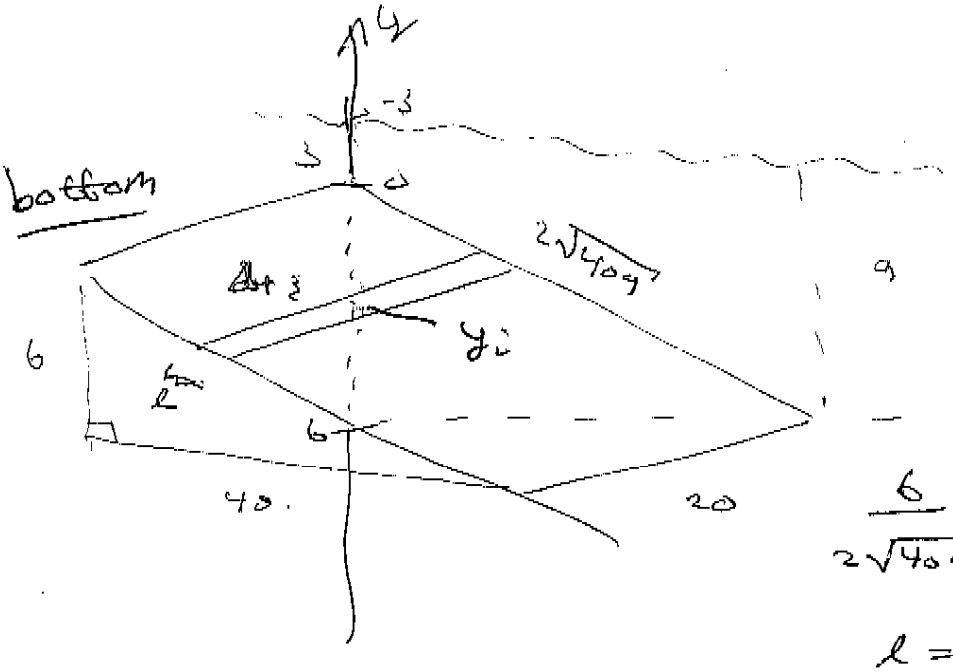
5

d) bottom



$$\frac{6}{2\sqrt{409}} = \frac{\Delta y}{\ell}$$

$$\ell = \frac{\sqrt{409}}{3}\Delta y$$

$$A_i = \frac{20}{3}\sqrt{409}\,\Delta y$$

$$p_i = 62.5(3 + y_i)$$

$$F = \int_0^6 62.5(3+y)\left(\frac{20}{3}\right)\sqrt{409}\,dy$$

$$= 303356.2 \text{ lb}$$

8.3
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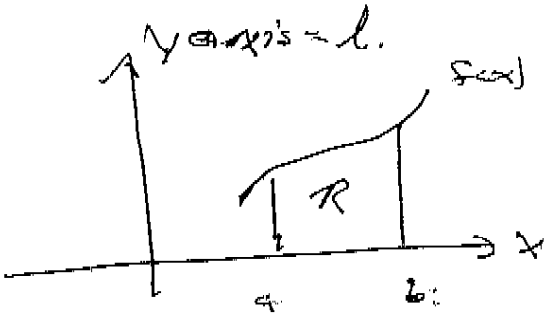
Recall $\bar{x} = \dfrac{\int_a^b x f(x)\,dx}{\int_a^b f(x)\,dx}$

$= \frac{1}{A} \int_a^b x f(x)\,dx$

$\Rightarrow \bar{x} A = \int_a^b x f(x)\,dx$.

Theorem of Pappus.

Let $\mathcal{R}$ be a plane region that lies entirely on one side of a line l in the plane. If $\mathcal{R}$ is rotated about l, then the volume of the resulting solid is the product of the area A of $\mathcal{R}$ and the distance d traveled by the centroid of $\mathcal{R}$.



□ proof (special case).

using cylindrical shells

$V = \int_a^b 2\pi x f(x)\,dx$

$= 2\pi \int_a^b x f(x)\,dx$

$= 2\pi \bar{x} A$.

$= A d$ ▣

Ex3: Find the volume of a sphere w/ radius r.

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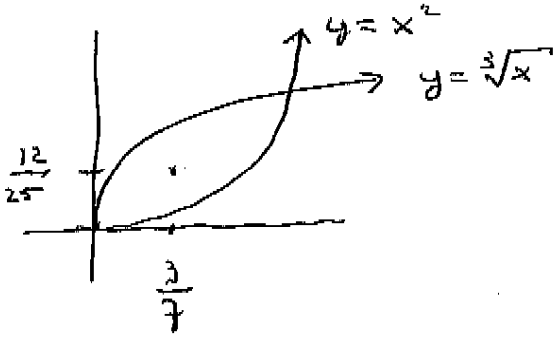
The region bounded by $y=x^2$ & $y=\sqrt[3]{x}$ is rotated about the x-axis ... find the volume ... of the solid.

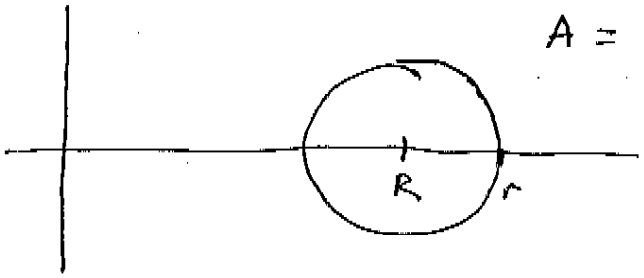


$A = \frac{5}{12}$

$V = \frac{5}{12} \cdot 2\cdot\pi \cdot \frac{12}{25}$

$= \frac{2}{5}\pi.$

Find the volume of the torus.



$A = \pi r^2$ and $V = 2\pi R \cdot \pi r^2$

$= 2\pi^2 R r^2$

8.3
7/7

A cylindrical tank w/ axis that is horizontal & w/ radius 4m is filled w/ gasoline (density 672 kg/m³). What is the total force on one end of this full tank?