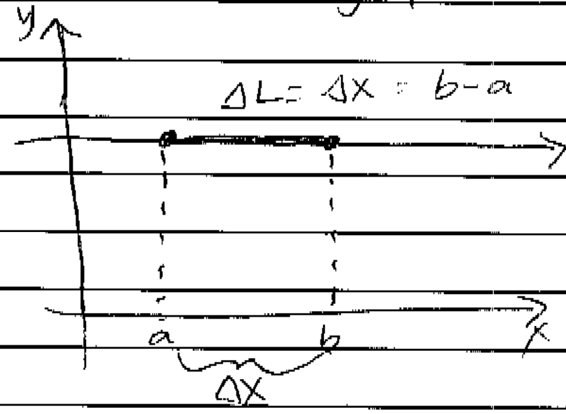
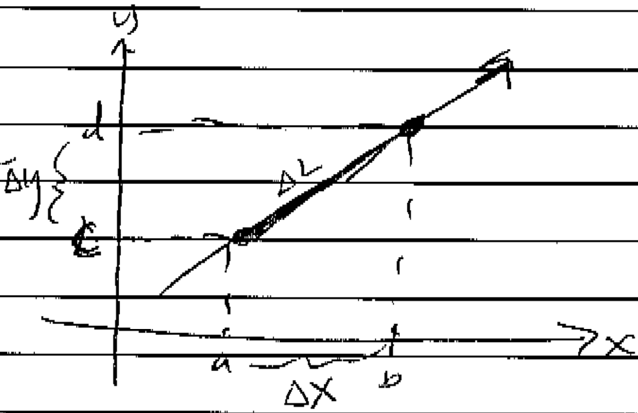
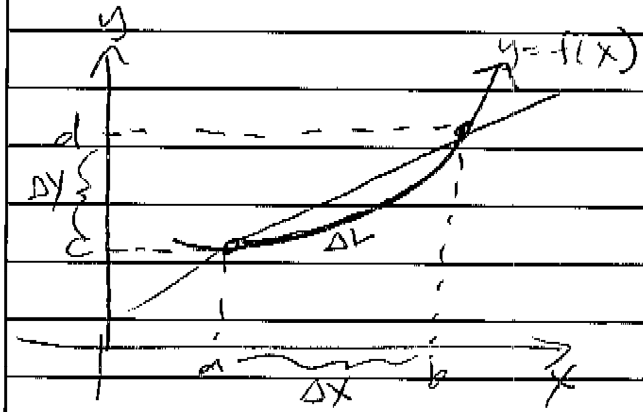
Notetaker's name Emma Miller Class CALC 125 Date 5/26/09

8.1 - Arc Length





$$\Delta L = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$



$$\Delta L \approx \sqrt{(\Delta x)^2 + (\Delta y)^2}$$
$$= \sqrt{\Delta x^2 \left(1 + \left(\frac{\Delta y}{\Delta x}\right)^2\right)}$$

$$\Delta L \approx \Delta x \sqrt{1 + \left(\Delta y / \Delta x\right)^2}$$

As $\Delta x \mapsto 0$ $\Delta y / \Delta x \rightarrow$ slope of tangent
↳ derivative $\frac{dy}{dx}$

so, as $\Delta x \rightarrow 0$
$$dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

and the arc length of $y = f(x)$ on $a \le x \le b$ is:
$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \qquad L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

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Example 1: Find the arclength of $y = \frac{1}{3}(x^2+2)^{3/2}$ on $[0,1]$

Need: $\frac{dy}{dx} = \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{\cancel{2}}(x^2+2)^{1/2} \cdot \cancel{2}x$

$$L = \int_0^1 \sqrt{1+(x\sqrt{x^2+2})^2}\,dx$$

$$= \int_0^1 \sqrt{1+x^2(x^2+2)}\,dx$$

$$= \int_0^1 \sqrt{1+x^4+2x^2}\,dx \qquad x^4+2x^2+1 = (x^2+1)^2$$

$$= \int_0^1 (x^2+1)\,dx$$

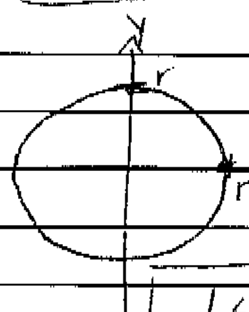
$$= \left[\frac{x^3}{3}+x\right]_0^1 = \frac{4}{3}$$

Example 2: Find arclength of $y = \sin x$ on $[0, \pi]$

need: $dy/dx = \cos x$ $\quad L = \int_0^\pi \sqrt{1+(\cos x)^2}\,dx$

↳ can't solve! stuck

Example 3: Derive circumference of a circle w/ radius r



$y = \sqrt{r^2-x^2}$ on $[0,r]$ $\qquad (C = 2\pi r)$

$y' = \frac{1}{2}(r^2-x^2)^{-1/2} \cdot (2x)$

$\Rightarrow y' = \frac{-x}{\sqrt{r^2-x^2}}$ $\qquad C = 4\int_0^r \sqrt{1+\left(\frac{-x}{\sqrt{r^2-x^2}}\right)^2}\,dx$

$$= 4\int_0^r \sqrt{1+x^2/(r^2-x^2)}\,dx$$

$$4\int_0^r \sqrt{\frac{r^2-x^2+x^2}{r^2-x^2}}\,dx$$

$$4r\int_0^r \frac{dx}{\sqrt{r^2-x^2}}$$

$$\left[4r\arcsin\left(\frac{x}{r}\right)\right]_0^r$$

$$= 4r(\sin^{-1}(1) - \sin^{-1}(0))$$

$$4r(\pi/2 - 0)$$

$$= 2\pi r$$

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recall $\Delta L \approx \sqrt{(\Delta x)^2 + (\Delta y)^2}$

$= \sqrt{(\Delta y)^2\left(1 + (\Delta x/\Delta y)^2\right)}$

$= \Delta y\sqrt{1 + (\Delta x/\Delta y)^2}$

As $\Delta y \to 0$, $\Delta x/\Delta y \to dx/dy$

so, the arclength of $x = g(y)$ on $c \le x \le d$ is

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

Example 4: Find arclength of $x = \frac{1}{3}\sqrt{y}(y-3)$ on $[0,9]$

$\frac{1}{3}y^{3/2} - y^{1/2}$

need: $\frac{dx}{dy} = \frac{1}{2}y^{1/2} - \frac{1}{2\sqrt{y}}$

$$L = \int_0^9 \sqrt{1 + \left(\frac{\sqrt{y}}{2} - \frac{1}{2\sqrt{y}}\right)^2}\, dy$$

$$= \int_0^9 \sqrt{1 + \left(\frac{y}{4} - \frac{1}{2} + \frac{1}{4y}\right)}\, dy$$

$$= \int_0^9 \sqrt{\frac{4y + y^2 - 2y + 1}{4y}}\, dy$$

$$= \int_0^9 \sqrt{\frac{y^2 + 2y + 1}{4y}}\, dy$$

$$= \int_0^9 \frac{y+1}{2\sqrt{y}}\, dy = \frac{1}{2}\int_0^9 \left(\sqrt{y} + \frac{1}{\sqrt{y}}\right) dy$$

$$= \frac{1}{2}\left[\frac{2}{3}y^{3/2} + 2y^{1/2}\right]_0^9 = \frac{1}{2}\left(\frac{2}{3}(27) + 6\right) = 12$$