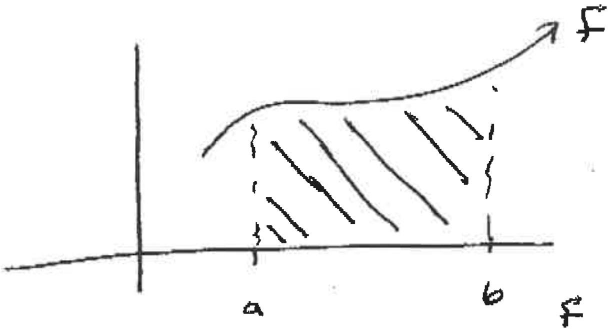
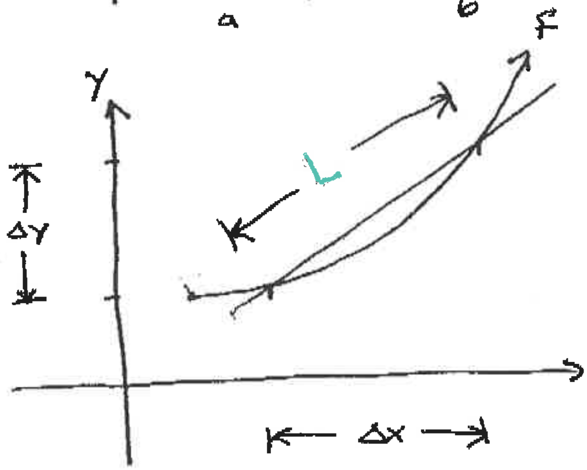
Arclength

We can find the area under the curve.



What about the length of the curve?



$$\Delta L = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$\Delta L = \sqrt{(\Delta x)^2 + \frac{(\Delta y)^2 (\Delta x)^2}{(\Delta x)^2}}$$

$$\Delta L = \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}\ \Delta x$$

Now $\lim_{\Delta x \to 0} \sqrt{1+\left(\frac{\Delta y}{\Delta x}\right)^2}\ \Delta x = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\ dx$

So, it seems reasonable that if f' is continuous on $[a,b]$, then the length of the curve $y = f(x)$ on $[a,b]$ is

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\ dx$$

Ex 1: Find the length of the curve $y = \frac{1}{3}($

on $[0,1]$.
$$\int_0^1 \sqrt{1+\left(\tfrac{1}{2}(x^2+2)^{1/2}\cdot 2x\right)^2}\,dx$$
$$= \int_0^1 \sqrt{1+x^2(x^2+2)}\,dx$$

Ex 2: Find the arclength of $x = \frac{1}{3}\sqrt{y}(y-3)$

$0 \leq y \leq 9$. (must play w/ the algebra).

Ex 3: Find the arclength of $y = x^2$ on

(use trig substitution).