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7.7: Numerical Integration

Face it, finding antiderivatives is a pain.

Ex 1: $\int_0^{1} \sin(x^2)\,dx = \sqrt{\frac{\pi}{2}}$ FresnelS $\left[\sqrt{\frac{2}{\pi}}\right]$

≈ 0.3102683

a) L_{10}

b) R_{10}

c) M_{10}

Ex 2: $\int_0^2 e^{-x^2}\,dx = \sqrt{\frac{\pi}{4}}$ Erf[1]

≈ 0.7468241

a) L_8, R_8, and M_8.

~~Ex3:~~ The Trazoidal Rule.

$$T_n = \frac{L_n + R_n}{2} = \frac{f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-1}) + f(x_n)}{2}$$

Ex3: a) approx $\int_0^1 \sin(x^2)\,dx$ w/ T_{10}

b) approx $\int_0^2 e^{-x^2}\,dx$ w/ T_8.

Simpson's Rule

The previous approx techniques use lines to approx curves. An alternative is to approx w/ quadratics taking 2 pts at a time.

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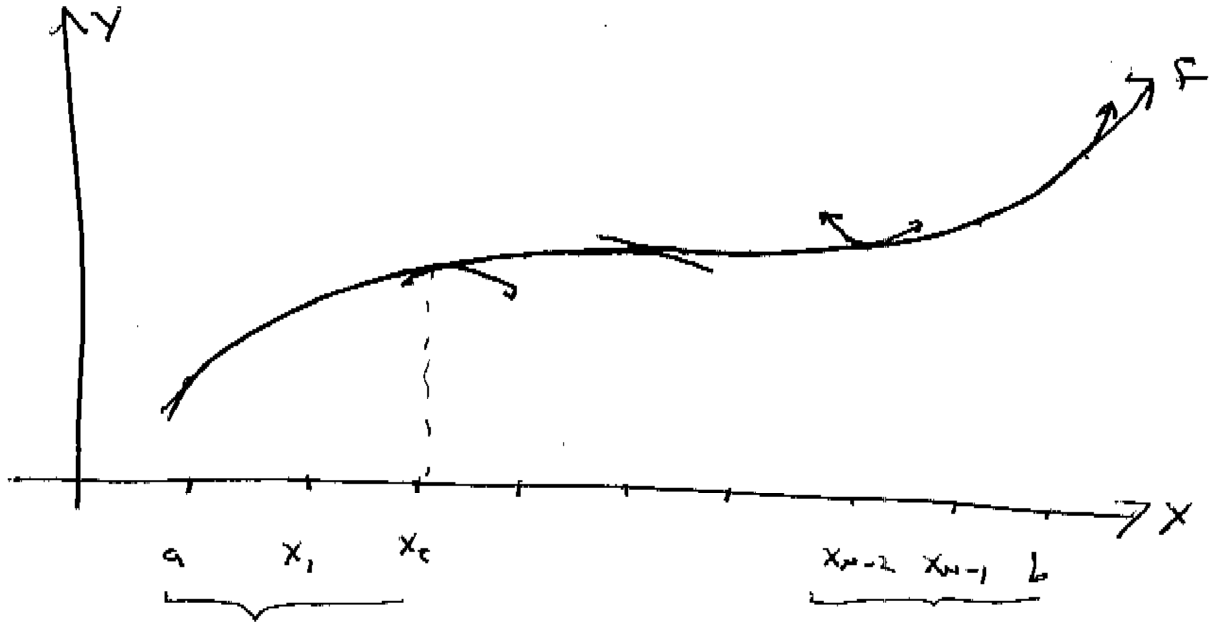

$$\frac{\Delta x}{3}(f(x_0)+4f(x_1)+f(x_2)) + \dots + \frac{\Delta x}{3}(f(x_{n-2})+4f(x_{n-1})+f(x_n))$$

$$= \frac{\Delta x}{3}\left[f(x_0)+4(f(x_1))+2f(x_2)+4(f(x_3))+\dots+4f(x_{n-1})+f(x_n)\right]$$

w/ coefficients 1, 4, 2, 4, 2, ... , 4, 1.

Ex4: a) approx $\int_0^1 \sin(x^2)\,dx$ w/ S_{10}

b) approx $\int_0^2 e^{-x^2}\,dx$ w/ S_8.

Note that $S_{2N} = \frac{1}{3}T_N + \frac{2}{3}M_N$

Ex5: a) approx $\int_0^1 \sin(x^2)\,dx$ w/ S_{20}

b) approx $\int_0^2 e^{-x^2}\,dx$ w/ S_{16}.

Tomorrow: Errorbounds.

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ex 6; Approx ~~Use~~ $\int_0^{2} \frac{\cancel{\sin(x^2)}}{e^{3x}}\,dx$ using T_8 & S_8

Error Bounds : suppose $|f''(x)| \le k$ for $a \le x \le b$. If E_T & E_m are the errors in the Trap & Midpt Rules, then...

$$|E_T| \le \frac{k(b-a)^3}{12n^2} \quad \& \quad |E_m| \le \frac{k(b-a)^3}{24n^2}$$

Suppose that $|f^{(4)}(x)| \le k$ for $a \le x \le b$, If E_s is the error involved in using Simpson's Rule, then

$$|E_s| \le \frac{k(b-a)^5}{180n^4}$$

ex 6 rev: (A) compare error bounds. for T_8 & S_8.

(B) How large an n must we choose to guarantee the approx is w/in 0.00001 of the exact value.

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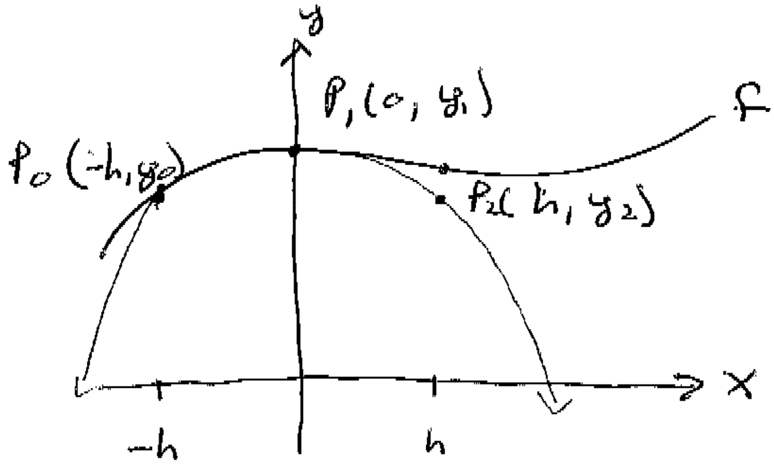
Simpson's Rule: uses quadratics ... two subintervals @ a time.



Area under the parabola $\int_{-h}^{h} (Ax^2 + Bx + C)\,dx$

$$= 2\int_0^h (Ax^2 + C)\,dx$$

$$= 2\left[\frac{A}{3}x^3 + Cx\right]_0^h$$

$$= 2\left(\frac{A}{3}h^3 + Ch\right)$$

$$= \frac{h}{3}\left(2Ah^2 + 6C\right)$$

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but, the parabola passes thru.

$(-h, y_0)$, $(0, y_1)$, & (h, y_2)

$$y_0 = A(-h)^2 + B(-h) + c$$
$$= Ah^2 - Bh + c$$

$$y_1 = c$$

$$y_2 = Ah^2 + Bh + c$$

Add $y_0 + 4y_1 + y_2 = 2Ah^2 + 6c$

so, ... the area under the parabola

$$\frac{h}{3}(2Ah^2 + 6c) = \frac{h}{3}(y_0 + 4y_1 + y_2)$$

The pattern for simpson's Rule

1, 4, 2, 4, 2, ..., 4, 1