While labeled as section 8.3, these notes are from a previous edition of the text and are equivalent to section 7.3 in the current ETV of Stewart.

$\int \sqrt{a^2 - x^2}\, dx$

Let $x = a\sin(\theta)$ on $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$= \sqrt{a^2 - a^2\sin^2(\theta)}$

$= \sqrt{a^2(1 - \sin^2(\theta))}$

$= \sqrt{a^2\cos^2(\theta)}$

$= a|\cos(\theta)|$ but $\cos(\theta) \geq 0$ on $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$= a\cos(\theta)$.

Ex 1: $\int x^2\sqrt{4-x^2}\, dx$

Let $x = 2\sin(\theta) \Rightarrow dx = 2\cos(\theta)\,d\theta$.

$= \int 4\sin^2(\theta)\sqrt{4 - 4\sin^2(\theta)} \cdot 2\cos(\theta)\,d\theta$

$= \int 4\sin^2(\theta)\,(2\cos(\theta)) \cdot 2\cos(\theta)\,d\theta$

$= 16\int \sin^2(\theta)\cdot\cos^2(\theta)\,d\theta$.

$= 16\int \left(\frac{1-\cos(2\theta)}{2}\right)\left(\frac{1+\cos(2\theta)}{2}\right)d\theta$

$= 4\int\left[1 - \cos^2(2\theta)\right]d\theta$

$$\hookrightarrow = 4 \int \left[1 - \frac{1+\cos(4\theta)}{2}\right] d\theta$$

$$= 2 \int \left[2 - 1 - \cos(4\theta)\right] d\theta$$

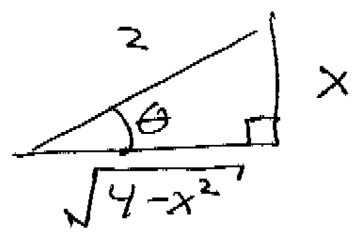
$$= 2 \int \left[1 - \cos(4\theta)\right] d\theta$$

$$= 2 \left[\theta - \frac{\sin(4\theta)}{4}\right] + C$$

$$= 2 \left[\theta - \frac{1}{4}\left(2\sin(2\theta)\cos(2\theta)\right)\right] + C$$

$$= 2 \left\{\theta - \frac{1}{4}\left(4\sin(\theta)\cos(\theta)\left(\cos^2(\theta) - \sin^2(\theta)\right)\right)\right\} + C$$

since $x = 2\sin(\theta) \Rightarrow \frac{x}{2} = \sin(\theta)$



$$= 2 \left[\sin^{-1}\left(\frac{x}{2}\right) + \frac{x}{2}\left(\frac{\sqrt{4-x^2}}{2}\right)\left(\frac{4-x^2}{4} - \frac{x^2}{4}\right)\right] + C$$

$\vdots$

To work w/ expressions...

expression	Sub	Identity
$\sqrt{a^2 - x^2}$	$x = a\sin(\theta)$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$	$1 - \sin^2(\theta) = \cos$
$\sqrt{a^2 + x^2}$	$x = a\tan(\theta)$ "	$1 + \tan^2 = \sec^2$
$\sqrt{x^2 - a^2}$	$x = a\sec(\theta)$, $0 \le \theta < \frac{\pi}{2}$ or $\pi \le \theta < \frac{3\pi}{2}$	$\sec^2 - 1 = \tan$

Ex 2: Find the area of a circle w/ radius a.

Ex 3: $\int x\sqrt{x^2 + 16}\, dx$

Ex 4: $\int \frac{1}{\sqrt{9x^2 + 6x - 8}}\, dx$ complete the square.

Ex 5: $\int \frac{dx}{(x^2 - 1)^{3/2}} = \int \frac{dx}{(\sqrt{x^2 - 1})^3}$ Let $x = \sec(\theta)$.

$dx = \sec(\theta)\tan(\theta)\, d\theta$

$$= \int \frac{\sec(\theta)\tan(\theta)}{(\sqrt{\sec^2(\theta) - 1})^3}\, d\theta$$

$$= \int \frac{\sec(\theta)\tan(\theta)}{\tan^3(\theta)}\, d\theta.$$

$$= \int \frac{\sec(\theta)}{\tan^2(\theta)}\, d\theta = \int \frac{\cos^2(\theta)}{\sin^2(\theta)\cos(\theta)}\, d\theta.$$