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Section 7.2

Trig Integrals.

Ex: 1

$I = \int \sec^3(x)\,dx = \int \sec(x)\cdot\sec^2(x)\,dx$

$= \sec(x)\tan(x) - \int \sec(x)\underbrace{\tan^2(x)}_{\sec^2(x)-1}\,dx$

$= \sec(x)\tan(x) - \int \sec^3(x) - \sec(x)\,dx.$

$= \sec(x)\tan(x) - \int \sec^3 x^{dx} + \int \sec(x)^{dx}.$

$2\int \sec^3(x)\,dx = \sec(x)\tan(x) + \int \sec(x)\,dx$

Parts:

$u = \sec(x) \qquad dv = \sec^2(x)\,dx$

$du = \sec(x)\tan(x)\,dx \qquad v = \tan(x)$

⇒ $2\int \sec^3(x)\,dx = \sec(x)\tan(x) + \ln|\sec(x) + \tan(x)| + C$

$\int \sec^3(x)\,dx = \frac{1}{2}(\sec(x)\tan(x) + \ln|\sec(x)+\tan(x)|) + C$

Sub problem:

$\int \sec(x)\,dx = \underbrace{x\sec(x) - \int x\sec(x)\tan(x)}$

does not work gets more complicate

$u = \sec(x) \qquad dv = dx$

$du = \sec(x)\tan(x)\,dx \qquad v = x.$

$\int \sec(x)\,dx = \int \sec(x)\cdot\frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}\,dx$

$= \int \frac{\sec^2(x) + \sec(x)\tan(x)}{\sec(x)+\tan(x)}\,dx$

let $t = \sec(x) + \tan(x)$

⇒ $dt = \sec(x)\tan(x) + \sec^2(x)\,dx$

$= \int \frac{1}{u}\,du$

$= \ln|u| + C$

$= \ln|\sec(x) + \tan(x)| + C.$

Ex: 2

$\sin^2(x) = 1 - \cos^2(x)$

Substitution:

let $u = \cos(x)$

$du = -\sin(x)\,dx$

$\int \sin^5(x)\cos^2(x)\,dx = \int \sin(x)\cdot\overbrace{\sin^4(x)}^{(\sin^2(x))^2}\cdot\cos^2(x)\,dx$

$= \int (1-\cos^2(x))^2\cos^2(x)\cdot\sin(x)\,dx$

$= -\int (1-u^2)^2\cdot u^2\,du = -\int (1 - 2u^2 + u^4)\,u^2\,du$

$= -\int u^2 - 2u^4 + u^6\,du$

$= -\left(\frac{1}{3}u^3 - \frac{2}{5}u^5 + \frac{1}{7}u^7\right) + C$

$= -\left(\frac{1}{3}\cos^3(x) - \frac{2}{5}\cos^5(x) + \frac{1}{7}\cos^7(x)\right) + C.$

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Ex: 3

$$\int \cos^2(x)\, dx = \int \frac{1+\cos(2x)}{2}\, dx$$

$$= \tfrac{1}{2}x + \frac{\sin(2x)}{4} + C.$$

half angle identity

$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$

$$\sin^2(x) = \frac{1-\cos(2x)}{2}.$$

Ex: 4

$$\int \sin^4(x)\, dx = \int \left(\sin^2(x)\right)^2 dx = \int \left(\frac{1-\cos(2x)}{2}\right)^2 dx$$

$$= \frac{1}{4}\int \left(1 - 2\cos(2x) + \cos^2(2x)\right) dx$$

$$= \tfrac{1}{4}\int \left(1 - 2\cos(2x) + \frac{1+\cos 4x}{2}\right) dx$$

$$= \tfrac{1}{4}\left(x - \sin(2x) + \tfrac{1}{2}x + \frac{\sin(4x)}{8}\right) + C.$$