part A

100	90's	80's	70's	60's	< 60
7	9	7	4	2	3

Test 1 – Part A
Dusty Wilson
Math 220

overall
high 100%
$\bar{x} = 79.4\%$
med = 80.4%

Name: Key

You know that I write slowly. This is chiefly because I am never satisfied until I have said as much as possible in a few words, and writing briefly takes far more time than writing at length.

Johann Carl Friedrich Gauss
1777-1855 (German mathematician)

No work = no credit
No Graphing Calculators

Warm-ups (1 pt each): $\vec{e}_1 + \vec{e}_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $\vec{e}_1 \cdot \vec{e}_3 = 0$ $\vec{e}_2 \cdot \vec{e}_2 = 1$

1.) (1 pts) According to Gauss (above), why did he write slowly? Answer using complete English sentences.

Gauss was very concise,

2.) (6 pts) True or False

T or F
a.) If A is a 3 x 4 matrix and vector $\vec{v} \in \mathbb{R}^4$, the vector $A\vec{v} \in \mathbb{R}^3$

$$A_{3\times4}\,\vec{v}_{4\times1} = (A\vec{v})_{3\times1}$$

T or F
b.) If matrix A is in rref, then at least one of the entries in each column must be 1.

$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$ is in rref

T or F
c.) $\text{rank}\begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix} = 2$

$\text{rref}\left(\begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}\right) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ so the rank = 1

3.) (4 pts) What are four applications of matrices?

computer graphics.
animation
Google Page rank.
Encryption,
Graph Theory (traffic example)
Statics (solving systems).
regression (Ceres story)

4.) (4 pts) Use matrices to solve the system $\begin{cases} 4x+3y=1 \\ x+2y=4 \end{cases}$

$$\left[\begin{array}{cc|c} 4 & 3 & 1 \\ 1 & 2 & 4 \end{array}\right] \quad R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{cc|c} 1 & 2 & 4 \\ 4 & 3 & 1 \end{array}\right] \quad R_2 - 4R_1 \to R_2$$

$$\left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & -5 & -15 \end{array}\right] \quad -\frac{1}{5}R_2 \to R_2$$

$$\to \left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & 1 & 3 \end{array}\right] \quad R_1 - 2R_2 \to R_2$$

$$\left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & 3 \end{array}\right]$$

$x = -2$
$y = 3$

5.) (4 pts) Use Gauss-Jordan Elimination to solve the system with augmented matrix $\left[A \mid \vec{b}\right]$

$\left[\begin{array}{ccc|c} 2 & -1 & 7 & 33 \\ 1 & 0 & -2 & -5 \\ 0 & 2 & 2 & 10 \end{array}\right]$. Express your answer in vector form. $R_1 \leftrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & -5 \\ 2 & -1 & 7 & 33 \\ 0 & 2 & 2 & 10 \end{array}\right] \quad R_2 - 2R_1 \to R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & -5 \\ 0 & -1 & 11 & 43 \\ 0 & 2 & 2 & 10 \end{array}\right] \quad -R_2 \to R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & -5 \\ 0 & 1 & -11 & -43 \\ 0 & 2 & 2 & 10 \end{array}\right] \quad R_3 - 2R_2 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & -5 \\ 0 & 1 & -11 & -43 \\ 0 & 0 & 24 & 96 \end{array}\right] \quad \frac{1}{24}R_3 \to R_3$$

$$\to \left[\begin{array}{ccc|c} 1 & 0 & -2 & -5 \\ 0 & 1 & -11 & -43 \\ 0 & 0 & 1 & 4 \end{array}\right] \quad R_2 + 11R_3 \to R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & -5 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 4 \end{array}\right] \quad R_1 + 2R_3 \to R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$$\vec{x} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$$

6.) (4 pts) The reduced row echelon form of an augmented system $A\bar{x}=\bar{b}$ is

$$\text{rref}\left(\left[A\,|\,\bar{b}\right]\right)=\left[\begin{array}{ccccc|c} 1 & -2 & 0 & 0 & 2 & 3 \\ 0 & 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$
$$\begin{array}{ccccc} x_1 & r & x_3 & x_4 & t \end{array}$$

. Write $\bar{x}$ in vector form.

$$\vec{x}=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}=\begin{bmatrix} 3+2r-2t \\ r \\ 2+t \\ -4-3t \\ t \end{bmatrix}=\begin{bmatrix} 3 \\ 0 \\ 2 \\ -4 \\ 0 \end{bmatrix}+r\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}+t\begin{bmatrix} -2 \\ 0 \\ 1 \\ -3 \\ 1 \end{bmatrix}$$

7.) (4 pts) The vector $\begin{bmatrix} 2-3t \\ t-5 \\ t \end{bmatrix}$ is the solution to what matrix in rref?

$$\vec{x}=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 2-3t \\ t-5 \\ t \end{bmatrix} \Rightarrow \begin{array}{l} x_1 = 2-3x_3 \\ x_2 = x_3-5 \\ x_3 = \text{free} \end{array} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 3 & 2 \\ 0 & 1 & -1 & -5 \end{array}\right]$$

8.) (4 pts) Write the augmented matrix for a system of linear equations that is inconsistent.

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 0 & 1 \end{array}\right] \leftarrow \text{row w/ } [0 \cdots 0 \,|\, 1]$$

Test 1 – Part B
Dusty Wilson
Math 220

Name: ______________________________

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Johann Carl Friedrich Gauss
1777-1855 (German mathematician)

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1.) (6 pts) True or False (circle one).

a.) (True or False) There exists a 3 x 4 matrix with rank 4.

At most 1 pivot per row and only 3 rows.

b.) (True or False) If A is an n x n matrix and $\vec{x} \in \mathbb{R}^n$, then the product $A\vec{x}$ is a linear combination of the columns of matrix A.

$$A\vec{x} = \begin{bmatrix} | & & | \\ \vec{v}_1 & \cdots & \vec{v}_n \\ | & & | \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = x_1\vec{v}_1 + \cdots + x_n\vec{v}_n$$

c.) (True or False) If A and B are matrices of the same size, then the formula $\operatorname{rank}(A+B) = \operatorname{rank}(A) + \operatorname{rank}(B)$ must hold.

$$\operatorname{rank}\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = 2 \neq \operatorname{rank}\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) + \operatorname{rank}\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = 4.$$

2.) (4 pts) If A is an n x m matrix and $\vec{x}, \vec{y} \in \mathbb{R}^m$, prove $A(\vec{x}+\vec{y}) = A\vec{x} + A\vec{y}$.

proof.

Let A, $\vec{x}$, and $\vec{y}$ be given as defined above.

$$A(\vec{x}+\vec{y}) = \begin{bmatrix} | & & | \\ \vec{v}_1 & \cdots & \vec{v}_m \\ | & & | \end{bmatrix} \left(\begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} + \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix} \right)$$

$$= \begin{bmatrix} | & & | \\ \vec{v}_1 & \cdots & \vec{v}_m \\ | & & | \end{bmatrix} \begin{bmatrix} x_1 + y_1 \\ \vdots \\ x_m + y_m \end{bmatrix}$$

$$= (x_1 + y_1)\vec{v}_1 + \cdots + (x_m + y_m)\vec{v}_m$$

$$= [x_1\vec{v}_1 + \cdots + x_m\vec{v}_m] + [y_1\vec{v}_1 + \cdots + y_m\vec{v}_m]$$

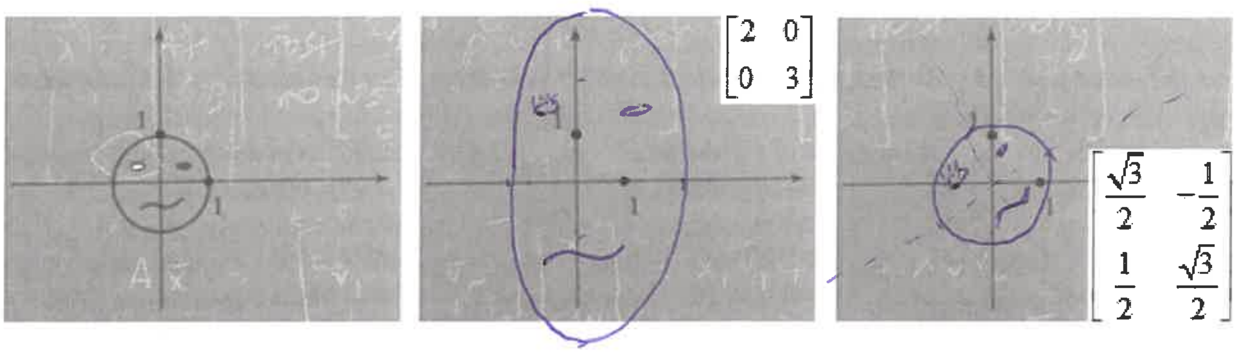
$$= \begin{bmatrix} | & & | \\ \vec{v}_1 & \cdots & \vec{v}_m \\ | & & | \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} + \begin{bmatrix} | & & | \\ \vec{v}_1 & \cdots & \vec{v}_m \\ | & & | \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix} = A\vec{x} + A\vec{y}$$

Q.E.D.

~~3.) (4 pts) Something about linear combinations~~

Hit w/ proof + T/F + parallelogram grid.

4.) (6 pts) Consider the circular face. Draw a sketch showing the effect of the linear transformation $T(\vec{x}) = A\vec{x}$ on this face. Make sure to clearly indicate the scale.



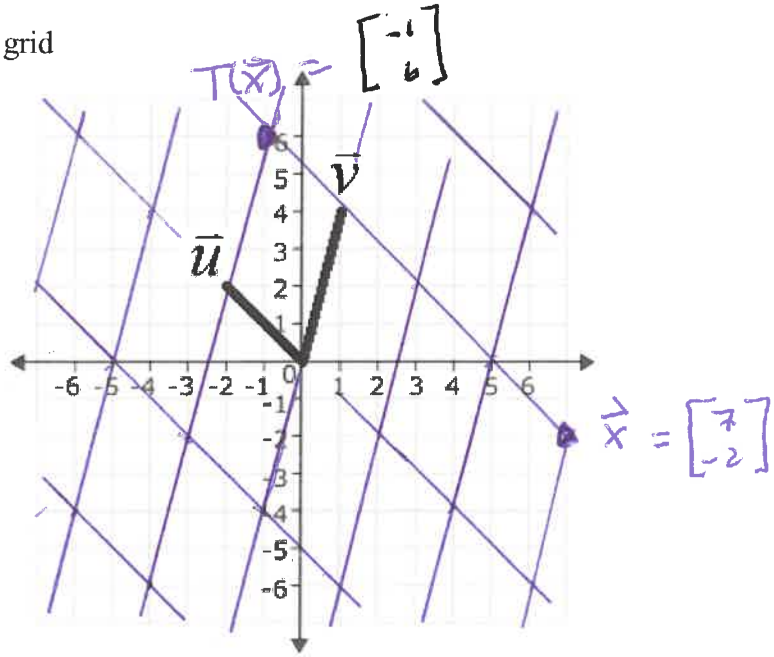
$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\sin \frac{\pi}{6} = \frac{\pi}{6}$

5.) (6 pts) Parallelogram grid



a.) Clearly graph and label the point $\vec{x} = \begin{bmatrix} 7 \\ -2 \end{bmatrix}$ on the grid.

b.) Find $\vec{x}$ in terms of the vectors $\vec{u}, \vec{v}$.

$\vec{x} = -3\vec{u} + \vec{v}$

c.) If T is a linear transformation such that $T(\vec{u}) = \vec{v} - \vec{u}$ and $T(\vec{v}) = 4\vec{v} - 2\vec{u}$, clearly graph and label the point $T(\vec{x})$.

$$T(\vec{x}) = T(-3\vec{u} + \vec{v}) = -3T(\vec{u}) + T(\vec{v})$$
$$= -3(\vec{v} - \vec{u}) + (4\vec{v} - 2\vec{u})$$
$$= -3\vec{v} + 3\vec{u} + 4\vec{v} - 2\vec{u}$$
$$= \vec{u} + \vec{v} = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$