Test 3a
Dusty Wilson
Math 220

No work = no credit
No Calculators

Name: KEY

Thus, the task is, not so much to see what no one has yet seen; but to think what nobody has yet thought, about that which everybody sees.

Erwin Rudolf Josef Alexander Schrödinger
1887 – 1961 (Austrian physicist)

1.) (10 pts) Find the QR factorization of $A = \begin{bmatrix} 3 & 35 \\ 4 & 55 \\ 0 & 0 \\ 0 & 12 \end{bmatrix}$

$$\begin{array}{cc} \vec{v}_1 & \vec{v}_2 \end{array}$$

$$\vec{u}_1 = \frac{\vec{v}_1}{\|\vec{v}_1\|} = \frac{1}{5}\begin{bmatrix} 3 \\ 4 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3/5 \\ 4/5 \\ 0 \\ 0 \end{bmatrix}$$

$$\vec{v}_2^{\perp} = \vec{v}_2 - (\vec{u}_1 \cdot \vec{v}_2)\vec{u}_1 = \begin{bmatrix} 35 \\ 55 \\ 0 \\ 12 \end{bmatrix} - 65\begin{bmatrix} 3/5 \\ 4/5 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \\ 0 \\ 12 \end{bmatrix}$$

$$\vec{u}_2 = \frac{\vec{v}_2^{\perp}}{\|\vec{v}_2^{\perp}\|} = \frac{1}{13}\begin{bmatrix} -4 \\ 3 \\ 0 \\ 12 \end{bmatrix} = \begin{bmatrix} -4/13 \\ 3/13 \\ 0 \\ 12/13 \end{bmatrix}$$

$$\Rightarrow \quad Q = \begin{bmatrix} 3/5 & -4/13 \\ 4/5 & 3/13 \\ 0 & 0 \\ 0 & 12/13 \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} 5 & 65 \\ 0 & 13 \end{bmatrix}$$

2.) (10 pts) Given A, S, D, and $\vec{x}_0$, find a closed form for $\vec{x}(t) = A^t \vec{x}_0$.

$$\vec{x}_0 = \begin{bmatrix} -12 \\ 7 \end{bmatrix} \qquad D = \begin{bmatrix} 3 & 0 \\ 0 & 17 \end{bmatrix}$$

$$A = S D S^{-1} = \begin{bmatrix} -193 & -490 \\ 84 & 213 \end{bmatrix} \qquad S = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$\vec{x}(t) = A^t \vec{x}_0$$

$$= S D^t \underbrace{S^{-1} \vec{x}_0}$$

$$\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} -12 \\ 7 \end{bmatrix} = \begin{bmatrix} 13 \\ 11 \end{bmatrix}$$

$$= S \underbrace{\begin{bmatrix} 3^t & 0 \\ 0 & 17^t \end{bmatrix} \begin{bmatrix} 13 \\ 11 \end{bmatrix}}$$

$$= \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 13 \cdot 3^t \\ 11 \cdot 17^t \end{bmatrix}$$

$$= \begin{bmatrix} 65 \cdot 3^t - 77 \cdot 17^t \\ -26 \cdot 3^t + 33 \cdot 17^t \end{bmatrix}$$