

Test 2  
Dusty Wilson  
Math 220

No work = no credit  
No Calculators

Name: KEY

*... we can repudiate completely and which we can abandon without regret because one does not know what this pretended sign signifies nor what sense one ought to attribute to it..*

Augustin Cauchy  
1789 – 1857 (French mathematician)

1.) (10 pts) Find the determinant of  $B = \begin{bmatrix} 2 & 0 & 3 & 0 \\ 1 & 0 & 3 & 1 \\ 3 & 2 & 1 & 3 \\ 3 & 0 & 2 & 1 \end{bmatrix}$

$$\det(B) = -2 \begin{vmatrix} 2 & 3 & 0 \\ 1 & 3 & 1 \\ 3 & 2 & 1 \end{vmatrix}$$

$$= -2 \left( 2 \begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix} - 3 \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} \right)$$

$$= -2 \left( 2(1) - 3(-2) \right)$$

$$= -16$$

2.) (10 pts) Find the eigenvalues of  $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 3 & 6 \\ 0 & 2 & 2 \end{bmatrix}$ . (Hint: All eigenvalues are integers).

solve  $\det(A - \lambda I) = 0$

$$\begin{aligned} \Rightarrow \begin{vmatrix} 1-\lambda & 2 & 3 \\ 3 & 3-\lambda & 6 \\ 0 & 2 & 2-\lambda \end{vmatrix} &= (1-\lambda) \begin{vmatrix} 3-\lambda & 6 \\ 2 & 2-\lambda \end{vmatrix} - 3 \begin{vmatrix} 2 & 3 \\ 2 & 2-\lambda \end{vmatrix} \\ &= (1-\lambda) \left[ \underbrace{(3-\lambda)(2-\lambda) - 12}_{6 - 5\lambda + \lambda^2 - 12} \right] - 3 \left[ \underbrace{2(2-\lambda) - 6}_{4 - 2\lambda - 6} \right] \\ &= (1-\lambda)(\lambda^2 - 5\lambda - 6) - 3(-2 - 2\lambda) \\ &= \lambda^2 - 5\lambda - 6 - \lambda^3 + 5\lambda^2 + 6\lambda + 6 + 6\lambda \\ &= -\lambda^3 + 6\lambda^2 + 7\lambda \\ &= -\lambda(\lambda^2 - 6\lambda - 7) \\ &= -\lambda(\lambda - 7)(\lambda + 1) \end{aligned}$$

The eigenvalues are  $0, -1, 7$ .