Consider the data in the following table.

Planet	a Mean Distance from the Sun (in Astronomical Units)	D Period of Revolution (in Earth Years)
Mercury	0.387	0.241
Earth	1.000	1.000
Jupiter	5.203	11.86
Uranus	19.19	84.04
Pluto	39.53	248.6

Use the methods discussed in Exercise 39 to fit a power function of the form $D = ka^n$ to these data. Explain, in terms of Kepler's laws of planetary motion. Explain why the constant k is close to 1.

we need to linearize our model and data.

$D = ka^n \Rightarrow \ln D = \ln(ka^n)$

$= \ln k + n \ln a$

This allows us to generate A and $\vec{b}$, for example: Mercury: $\ln(0.241) = 1 \ln(k) + n \ln(0.387)$

row 1 of $\vec{b}$ — col 1 — col 2 of row 2 of A.

$$A = \begin{bmatrix} 1 & -0.95 \\ 1 & 0 \\ 1 & 1.65 \\ 1 & 2.95 \\ 1 & 3.68 \end{bmatrix} \quad \text{and} \quad \vec{b} = \begin{bmatrix} -1.42 \\ 0 \\ 2.47 \\ 4.43 \\ 5.52 \end{bmatrix}$$

$$(A^TA)^{-1}A^T\vec{b} \approx \begin{bmatrix} 0 \\ 1.5 \end{bmatrix} \Rightarrow \ln D = 0 + 1.5 \ln a$$

$$= 1.5 \ln a$$

$$\Rightarrow D = a^{1.5}$$

or $D^2 = a^3$ (Kepler's 3rd law).

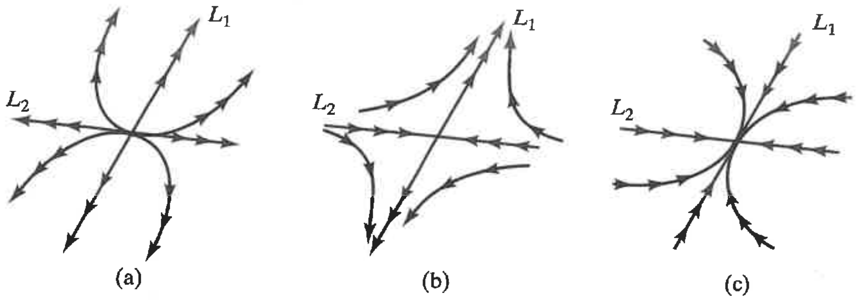

Figure 10 (a) $\lambda_1 > \lambda_2 > 1$. (b) $\lambda_1 > 1 > \lambda_2 > 0$. (c) $1 > \lambda_1 > \lambda_2 > 0$.

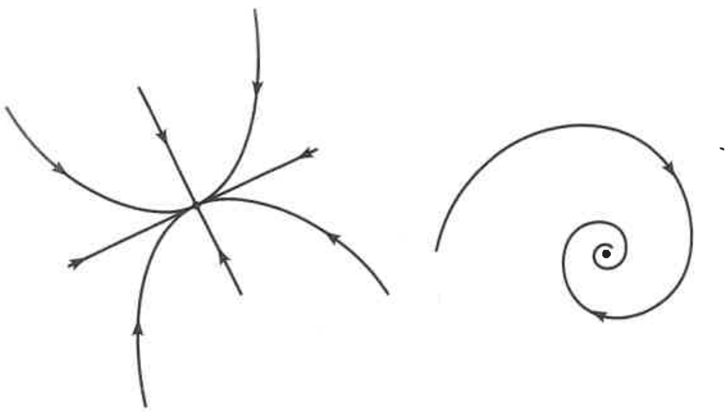

Figure 1(a) Asymptotically stable.

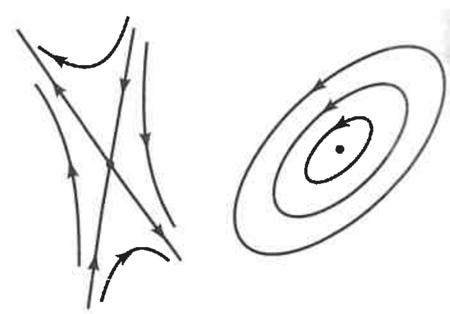

Figure 1(b) Not asymptotically stable.

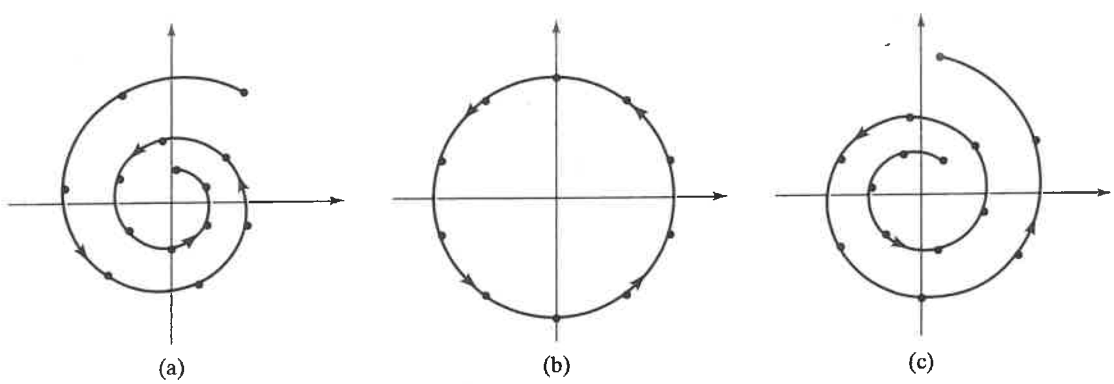

Figure 2 (a) $r < 1$: trajectories spiral inward. (b) $r = 1$: trajectories are circles. (c) $r > 1$: trajectories spiral outward.