

# 4.3: Matrix of a L.T.

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ex:  $T(f(t)) = f(2t-1)$  from  $P_2$  to  $P_2$

$$\left. \begin{aligned} T(1) &= 1 \\ T(t) &= 2t-1 \\ T(t^2) &= (2t-1)^2 = 4t^2 - 4t + 1 \end{aligned} \right\} \text{L.I.}$$

T is an isomorphism. ( $\text{im}(T) = P_2$  &  $\text{ker}(T) = 0$ )

$L_u$  is called the coordinate transformation from  $P_2$  to  $\mathbb{R}^3$

$$a + bt + ct^2 \xrightarrow{T} a + b(2t-1) + c(4t^2 - 4t + 1) = (a-b+c) + (2b-4c)t + 4ct^2$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\begin{bmatrix} a - b + c \\ 2b - 4c \\ 4c \end{bmatrix}$$

$B$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -4 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\begin{matrix} \uparrow & \uparrow & \uparrow \\ [T(1)]_u & [T(t)]_u & [T(t^2)]_u \end{matrix}$$

$$[T(t)]_u$$

must be in the same order as  $[\vec{x}]_B$ .

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ex:  $T(z) = (p+iq)z$  from  $\mathbb{C}$  to  $\mathbb{C}$ ,  $p, q \in \mathbb{R}$ .

$T(1) = (p+iq)1 = p+iq$   
 $T(i) = (p+iq)i = -q+pi$

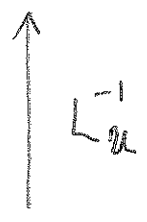
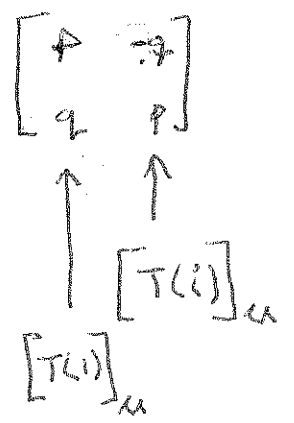
$\swarrow \searrow$   
 L.I.

$T$  is an isomorphism. ( $\text{im}(T) = \mathbb{C}$  and  $\text{ker}(T) = 0$ )

$z = a+bi \xrightarrow{(p+iq)z} (ap-bq) + i(aq+bp)$

$L_u$  is the coordinate transformation from  $\mathbb{C}$  to  $\mathbb{R}^2$ .

$[z]_u = \begin{bmatrix} a \\ b \end{bmatrix} \xrightarrow{B} \begin{bmatrix} ap-bq \\ aq+bp \end{bmatrix} = [T(z)]_u$



ex:  $T(M) = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} M$  from  $\mathbb{R}^{2 \times 2} \rightarrow \mathbb{R}^{2 \times 2}$

$$T\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$$

only two L.I. matrices so  $T$  is not an isomorphism

$$\text{im}(T) = \text{span}\left\{\begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}\right\}$$

$$\text{ker}(T) = \text{span}\left\{\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}\right\}$$

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

$L_u$  is a coordinate transformation from  $\mathbb{R}^{2 \times 2} \rightarrow \mathbb{R}^4$ .

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \xrightarrow{A} \begin{bmatrix} a+c & b+d \\ 2a+2c & 2b+2d \end{bmatrix}$$

$$\begin{array}{ccc} L_u \downarrow & & \downarrow L_u \\ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}_u & \xrightarrow{\quad} & \begin{bmatrix} a+c \\ b+d \\ 2a+2c \\ 2b+2d \end{bmatrix}_u \end{array}$$

$$B = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix}$$