with average value $\frac{25}{18}$ and amplitude $\sqrt{\frac{1}{18^2}+\frac{16}{27}} = \frac{\sqrt{193}}{18}$. Thus $M = \frac{25+\sqrt{193}}{18}$ and $m = \frac{25-\sqrt{193}}{18}$, so that the length of the semi-major axis of E is

$\sqrt{M} = \sqrt{\frac{25+\sqrt{193}}{18}} \approx 1.47$, and for the semi-minor axis we get

$\sqrt{m} = \sqrt{\frac{25-\sqrt{193}}{18}} \approx 0.79$.

True or False

Ch 6.TF.**1** T, by Theorem 6.2.3a, applied to the columns.

Ch 6.TF.**2** T, by Theorem 6.2.6.

Ch 6.TF.**3** T, By theorem 6.1.4, a diagonal matrix is triangular as well.

Ch 6.TF.**4** T, by Theorem 6.2.3b.

Ch 6.TF.**5** T, by Definition 6.1.1

Ch 6.TF.**6** F; We have $\det(4A) = 4^4 \det(A)$, by Theorem 6.2.3a.

Ch 6.TF.**7** F; Let $A = B = I_5$, for example

Ch 6.TF.**8** T; We have $\det(-A) = (-1)^6 \det(A) = \det(A)$, by Theorem 6.2.3a.

Ch 6.TF.**9** F; In fact, $\det(A) = 0$, since A fails to be invertible

Ch 6.TF.**10** F; The matrix A fails to be invertible if $\det(A) = 0$ by Theorem 6.2.4.

Ch 6.TF.**11** T; The determinant is 0 for $k = -1$ or $k = -2$, so that the matrix is invertible for all *positive* k.

Ch 6.TF.**12** F. There is only one pattern with a nonzero product, containing all the 1's. Since there are three inversions in this pattern, $\det A = -1$.

Ch 6.TF.**13** T. Without computing its exact value, we will show that the determinant is positive. The pattern that contains all the entries 100 has a product of $100^4 = 10^8$, with two inversions. Each of the other $4! - 1 = 23$ patterns contains at most two entries 100, with the other entries being less than 10, so that the product of each of these patterns is less than $100^2 \cdot 10^2 = 10^6$. Thus the determinant in more than $10^8 - 23 \cdot 10^6 > 0$, so that the matrix in invertible.

Ch 6.TF.**14** F; The correct formula is $\det(A^{-1}) = \frac{1}{\det(A^T)}$, by Theorems 6.2.1 and 6.2.8.

Ch 6.TF.**15** T; The matrix A is invertible.

Ch 6.TF.**16** T; Any nonzero noninvertible matrix A will do.

Ch 6.TF.17 T, by Theorem 6.2.7.

Ch 6.TF.18 F, by Theorem 6.3.1. The determinant can be -1.

Ch 6.TF.19 T, by Theorem 6.2.6.

Ch 6.TF.20 F; The second and the fourth column are linearly dependent.

Ch 6.TF.21 F; Note that $\det \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} = 2$.

Ch 6.TF.22 T, by Theorem 6.3.9.

Ch 6.TF.23 T, by Theorem 6.3.3, since $\|\vec{v}_i^{\perp}\| \leq \|\vec{v}_i\| = 1$ for all column vectors $\vec{v}_i$.

Ch 6.TF.24 T; We have $\det(A) = \det(\text{rref } A) = 0$.

Ch 6.TF.25 F; Let $A = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$, for example. See Theorem 6.2.10.

Ch 6.TF.26 F; Let $A = 2I_2$, for example

Ch 6.TF.27 T; Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$. The column vectors of A are orthogonal and they all have length 2.

Ch 6.TF.28 F; Let $A = \begin{bmatrix} 8 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, for example.

Ch 6.TF.29 F; In fact, $\det(A) = \det[\vec{u}\ \vec{v}\ \vec{w}] = -\det[\vec{v}\ \vec{u}\ \vec{w}] = -\vec{v} \cdot (\vec{u} \times \vec{w})$. We have used Theorem 6.2.3b and Definition 6.1.1.

Ch 6.TF.30 T; Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, for example.

Ch 6.TF.31 F; Note that $\det(S^{-1}AS) = \det(A)$ but $\det(2A) = 2^3(\det A) = 8(\det A)$.

Ch 6.TF.32 F; Note that $\det(S^TAS) = (\det S)^2(\det A)$ and $\det(-A) = -(\det A)$ have opposite signs.

Ch 6.TF.33 F; Let $A = 2I_2$, for example.

Ch 6.TF.34 F; Let $A = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, for example.

Ch 6.TF.**35** F; Let $A = I_2$ and $B = -I_2$, for example.

Ch 6.TF.**36** T; Note that $\det(B) = -\det(A) < \det(A)$, so that $\det(A) > 0$.

Ch 6.TF.**37** T; Let's do Laplace expansion along the first row, for example (see Theorem 6.2.10).

Then $\det(A) = \sum_{j=1}^{n}(-1)^{1+j}a_{1j}\det(A_{1j}) \neq 0$. Thus $\det(A_{1j}) \neq 0$ for at least one j, so that A_{1j} is invertible.

Ch 6.TF.**38** T; Note that $\det(A)$ and $\det(A^{-1})$ are both integers, and $(\det A)(\det A^{-1}) = 1$. This leaves only the possibilities $\det(A) = \det(A^{-1}) = 1$ and $\det(A) = \det(A^{-1}) = -1$.

Ch 6.TF.**39** T, since $\text{adj}(A) = (\det A)(A^{-1})$, by Theorem 6.3.9.

Ch 6.TF.**40** F; Note that $\det(A^2) = (\det A)^2$ cannot be negative, but $\det(-I_3) = -1$.

Ch 6.TF.**41** T; The product associated with the diagonal pattern is odd, while the products associated with all other patterns are even. Thus the determinant of A is odd, so that A is invertible, as claimed.

Ch 6.TF.**42** F; Let $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 5 & 2 \end{bmatrix}$, for example

Ch 6.TF.**43** T; Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. If $a \neq 0$, let $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$; if $b \neq 0$, let $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$; if $c \neq 0$, let $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, and if $d \neq 0$, let $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.

Ch 6.TF.**44** T; Use Gaussian elimination for the first column only to transform A into a matrix of the form

$$B = \begin{bmatrix} 1 & \pm1 & \pm1 & \pm1 \\ 0 & * & * & * \\ 0 & * & * & * \\ 0 & * & * & * \end{bmatrix}$$

Note that $\det(B) = \det(A)$ or $\det(B) = -(\det A)$. The stars in matrix B all represent numbers $(\pm1) \pm (\pm1)$, so that they are 2, 0, or -2. Thus the determinant of the 3×3 matrix M containing the stars is divisible by 8, since each of the 6 terms in Sarrus' rule is 8, 0 or -8. Now perform Laplace expansion down the first column of B to see that $\det(M) = \det(B) = +/-\det(A)$.

Ch 6.TF.**45** T; $A(\text{adj}A) = A(\det(A)A^{-1}) = \det(A)I_n = \det(A)A^{-1}A = \text{adj}(A)A$.

Ch 6.TF.**46** T; Laplace expansion along the second row gives $\det(A) = -k\det\begin{bmatrix} 1 & 2 & 4 \\ 8 & 9 & 7 \\ 0 & 0 & 5 \end{bmatrix} + C = 35k + C$, for some constant C (we need not compute that $C = -259$). Thus A is invertible except for $k = \frac{-C}{35}$ (which turns out to be $\frac{259}{35} = \frac{37}{5} = 7.4$).

Ch 6.TF.47 F; $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are both orthogonal and $\det(A) = \det(B) = 1$. However, $AB \neq BA$.