high = 101.6%
$\bar{x}$ = 73.4%
med = 78.1%

100+	90's	80's	70's	60's	< 60
1	3	6	5	7	5

Test 2
Dusty Wilson
Math 153

Name: Key

No work = no credit

No Symbolic Calculators

Notable enough, however, are the controversies over the series 1 - 1 + 1 - 1 + 1 - ... whose sum was given by Leibniz as 1/2, although others disagree. ... Understanding of this question is to be sought in the word "sum"; this idea, if thus conceived -- namely, the sum of a series is said to be that quantity to which it is brought closer as more terms of the series are taken -- has relevance only for convergent series, and we should in general give up the idea of sum for divergent series.

Leonard Euler (1707 - 1783)
Swiss mathematician

Warm-ups (1 pt each): $1+\frac{1}{2}+\frac{1}{3}+\ldots=$ ∞ $\qquad \sum_{n=1}^{\infty} 2\cdot\left(\frac{1}{3}\right)^n = \frac{\frac{2}{3}}{1-\frac{1}{3}} = 1$ $\qquad \sum_{n=1}^{\infty} 2 =$ ∞

1.) (1 pt) According to Euler, for what type of infinite series is it relevant to calculate the sum? Answer using complete English sentences.

Sums are only relevant for convergent series.

2.) (10 pts) Does $\sum_{n=2}^{\infty} \frac{5+n}{n\sqrt{n^3-1}}$ converge or diverge? Justify your answer.

L.C.T.

$$\lim_{n\to\infty} \frac{\frac{5+n}{n\sqrt{n^3-1}}}{\frac{1}{n^{3/2}}} = \lim_{n\to\infty} \frac{(5+n)\,n^{3/2}}{n\sqrt{n^3-1}} \cdot \frac{\frac{1}{n^{5/2}}}{\frac{1}{n^{5/2}}}$$

$$= \lim_{n\to\infty} \frac{\frac{5}{n}+1}{\sqrt{1-\frac{1}{n^{3}}}}$$

$$= 1$$

Since $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ is a convergent p-series,

we know $\sum_{n=2}^{\infty} \frac{5+n}{n\sqrt{n^3-1}}$ converges by the limit comparison test.

3.) (10 pts) Does $\sum_{n=1}^{\infty} \frac{(-1)^n n! \sqrt{n+2}}{4^n}$ diverge? If not, is it conditionally or absolutely convergent? Justify your answer.

ratio test.

$$\lim_{n \to \infty} \left| \frac{(-1)^{n+1} (n+1)! \sqrt{n+3}}{4^{n+1}} \cdot \frac{4^n}{(-1)^n n! \sqrt{n+2}} \right|$$

$$= \lim_{n \to \infty} \frac{(n+1)\sqrt{n+3}}{4\sqrt{n+2}}$$

$$= \infty > 1$$

Hence the series diverges by the ratio test.

4.) (10 pts) Does $\sum_{n=1}^{\infty} \frac{(-1)^n}{1+n^2}$ converge or diverge? If it converges, is it conditional or absolute convergence? Justify your answer.

consider $\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{1+n^2} \right| = \sum_{n=1}^{\infty} \frac{1}{1+n^2}$

$\lim_{n \to \infty} \frac{\frac{1}{1+n^2}}{\frac{1}{n^2}} = 1$. Since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is a convergent p-series, $\sum_{n=1} \frac{1}{1+n^2}$ converges by the L.C.T. and the original series $\sum_{n=1}^{\infty} \frac{(-1)^n}{1+n^2}$ is absolutely convergent.

5.) (10 pts) Determine whether the sequence $a_n = \left(\frac{n+2}{n}\right)^{-7n}$ converges or diverges. If it converges, find the limit.

$$\lim_{n\to\infty} \left(\frac{n+2}{n}\right)^{-7n} = \lim_{x\to\infty} \left(\frac{x+2}{x}\right)^{-7x}$$

$$= e^{\lim_{x\to\infty} -7x \ln\left(\frac{x+2}{x}\right)}$$

$$= e^{\lim_{x\to\infty} \frac{\ln\left(\frac{x+2}{x}\right)}{-\frac{1}{7x}}}$$

$$\overset{H}{=} e^{\lim_{x\to\infty} \frac{\frac{x}{x+2} \cdot \frac{x-(x+2)}{\cancel{x^2}}}{\frac{1}{7\cancel{x^2}}}}$$

$$= e^{\lim_{x\to\infty} \frac{-14x}{x+2}}$$

$$= e^{-14}$$

6.) (10 pts) Write $\vec{a}(2)$ in the form $\vec{a} = a_T\vec{T} + a_N\vec{N}$ without finding $\vec{T}$ and $\vec{N}$ for the position vector-valued function $\vec{r}(t) = t\vec{i} + t^2\vec{j} + t^3\ \vec{k}$ at $t = 2$. That is, you need to find a_T and a_N.

$$\vec{r}'(t) = \langle 1, 2t, 3t^2 \rangle \Big|_{t=2} \langle 1, 4, 12 \rangle$$

$$\vec{r}''(t) = \langle 0, 2, 6t \rangle \Big|_{t=2} \langle 0, 2, 12 \rangle$$

$$|\vec{r}'(2)| = \sqrt{1+16+144} = \sqrt{161}$$

$$a_T = \frac{\vec{r}' \cdot \vec{r}''}{|\vec{r}'|} = \frac{152}{\sqrt{161}}$$

$$a_N = \frac{|\vec{r}' \times \vec{r}''|}{|r'|} = \frac{|\langle 24, -12, 2 \rangle|}{\sqrt{161}} = \frac{\sqrt{724}}{\sqrt{161}}$$

So $\vec{a}(2) = \frac{152}{\sqrt{161}}\vec{T} + \frac{\sqrt{724}}{\sqrt{161}}\vec{N}$.

7.) (10 pts) The helix $\vec{r}(t) = \langle 3\cos t, 3\sin t, t\rangle$ has a constant curvature (why?). Find that curvature.

$\vec{r}'(t) = \langle -3s, 3c, 1\rangle$

$\Rightarrow \vec{r}''(t) = \langle -3c, -3s, 0\rangle$

$\Rightarrow |\vec{r}'| = \sqrt{9s^2 + 9c^2 + 1} = \sqrt{10}$

$\Rightarrow \vec{r}' \times \vec{r}'' = \langle +3s, -3c, 9\rangle$

$\Rightarrow |\vec{r}' \times \vec{r}''| = \sqrt{9s^2 + 9c^2 + 81} = 3\sqrt{10}$

$\Rightarrow \kappa = \dfrac{3\sqrt{10}}{(\sqrt{10})^3}$

$= \dfrac{3}{10}$

8.) (10 pts) Find the arclength of $\vec{r}(t) = \langle 3\cos t, 3\sin t, t\rangle$ on $0 \le t \le 2\pi$.

From above $|\vec{r}'(t)| = \sqrt{10}$

Arclength $= \int_0^{2\pi} \sqrt{10}\, dt$

$= 2\pi\sqrt{10}$

For an extra credit point, compare this to the arclength of $\vec{c}(t) = \langle 3\cos t, 3\sin t, 0\rangle$ on $0 \le t \le 2\pi$ and explain the similarity/difference geometrically. circumference. circle w/rad 3.

$2\pi \cdot 3$

The arclengths are similar, but the helix is longer since it also gains altitude.

Test II
Dusty Wilson
Math 153

Name: Key

No work = no credit

No Symbolic Calculators

Notable enough, however, are the controversies over the series 1 - 1 + 1 - 1 + 1 - ... whose sum was given by Leibniz as 1/2, although others disagree. ... Understanding of this question is to be sought in the word "sum"; this idea, if thus conceived -- namely, the sum of a series is said to be that quantity to which it is brought closer as more terms of the series are taken -- has relevance only for convergent series, and we should in general give up the idea of sum for divergent series.

Leonard Euler (1707 - 1783)
Swiss mathematician

Warm-ups (1 pt each): $\sum_{n=1}^{\infty} 0 = 0$ $\quad \sum_{n=1}^{\infty} 2\cdot\left(\frac{1}{7}\right)^n = \frac{2/7}{1-\frac{1}{7}} = \frac{1}{3}$ $\quad 1+\frac{1}{2}+\frac{1}{3}+\ldots = \infty$

Harmonic series

1.) (1 pt) According to Euler, for what type of infinite series is it relevant to calculate the sum? Answer using complete English sentences.

Sums only apply to convergent series.

One (1) exercise is optional (or 2 pts extra credit). Clearly indicate which you want graded as optional.

2.) (10 pts) Write $\vec{a}(3)$ in the form $\vec{a} = a_T\vec{T} + a_N\vec{N}$ without finding $\vec{T}$ and $\vec{N}$ for the position vector-valued function $\vec{r}(t) = t\vec{i} + t^2\vec{j} + t^3\,\vec{k}$ at $t = 3$. That is, you need to find a_T and a_N.

$\vec{r}\,'(t) = \langle 1, 2t, 3t^2\rangle \Big|_{t=3} \quad \langle 1, 6, 27\rangle \quad$ w/ $|\vec{r}\,'| = \sqrt{766}$

$\vec{r}\,''(t) = \langle 0, 2, 6t\rangle \Big|_{t=3} \quad \langle 0, 2, 18\rangle$

$\vec{r}\,' \cdot \vec{r}\,'' = 498$

$\vec{r}\,' \times \vec{r}\,'' = \langle 54, -18, 2\rangle \quad$ w/ $|\vec{r}\,' \times \vec{r}\,''| = \sqrt{3244}$

$a_T = \frac{\vec{r}\,' \cdot \vec{r}\,''}{|\vec{r}\,'|} = \frac{498}{\sqrt{766}}$

$a_N = \sqrt{\frac{3244}{766}}$

so $\vec{a}(3) = \frac{498}{\sqrt{766}}\vec{T} + \sqrt{\frac{3244}{766}}\vec{N}$

64

3.) (10 pts) Find the arclength of $\vec{r}(t) = \langle 4\cos t, 4\sin t, t \rangle$ on $0 \le t \le 2\pi$.

$\vec{r}'(t) = \langle -4s, 4c, 1 \rangle$

$\Rightarrow |\vec{r}'(t)| = \sqrt{16s^2 + 16c^2 + 1} = \sqrt{17}$

w/ the arclength

$$L = \int_0^{2\pi} \sqrt{17}\, dt$$

$$= 2\pi\sqrt{17}$$

For an extra credit point, compare this to the arclength of $\vec{c}(t) = \langle 4\cos t, 4\sin t, 0 \rangle$ on $0 \le t \le 2\pi$ and explain the similarity/difference geometrically. The arclength is the circumference of the circle w/ rad 4 $= 8\pi$.

This is a bit under $2\pi\sqrt{17}$ since the ends of the helix don't connect.

4.) (10 pts) The helix $\vec{r}(t) = \langle 4\cos t, 4\sin t, t \rangle$ has a constant curvature (why?). Find that curvature.

$\vec{r}' = \langle -4s, 4c, 1 \rangle$ w/ $|\vec{r}'| = \sqrt{17}$

$\vec{r}'' = \langle -4c, -4s, 0 \rangle$

$\Rightarrow \vec{r}' \times \vec{r}'' = \langle 4s, -4c, 16s^2 + 16c^2 \rangle$

$= \langle 4s, 4c, 16 \rangle$ w/ $|r' \times r''| = \sqrt{16 + 16^2} = \sqrt{272}$

so $\kappa = \dfrac{\sqrt{272}}{17\sqrt{17}} = \dfrac{4}{17}$

* 5.) (10 pts) Determine whether the <u>sequence</u> $a_n = \left(\frac{n}{n+3}\right)^{5n}$ converges or diverges. If it converges, find the limit.

$$\lim_{n\to\infty}\left(\frac{n}{n+3}\right)^{5n} = e^{\lim_{x\to\infty} 5x\ln\left(\frac{x}{x+3}\right)}$$

$$= e^{\lim_{x\to\infty}\frac{\ln x - \ln(x+3)}{\frac{1}{5x}}}$$

$$\overset{H}{=} e^{\lim_{x\to\infty}\frac{\frac{1}{x}-\frac{1}{x+3}}{-\frac{1}{5x^2}}}$$

$$= e^{\lim_{x\to\infty}\frac{\cancel{x}+3-\cancel{x}}{x(x+3)}\cdot\frac{-5x^2}{1}}$$

$$= e^{-15}$$

* 6.) (10 pts) Does $\sum_{n=2}^{\infty}\frac{5+n}{n\sqrt{n+1}}$ converge or diverge? Justify your answer.

LCT w/ $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ which is a divergent p-series.

$$\lim_{n\to\infty}\frac{\frac{5+n}{n\sqrt{n+1}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty}\frac{5+n}{n\sqrt{n+1}}\cdot\frac{\sqrt{n}}{1}\cdot\frac{\frac{1}{n^{3/2}}}{\frac{1}{n^{3/2}}}$$

$$= \lim_{n\to\infty}\frac{\frac{5}{n}+1}{1\sqrt{1+\frac{1}{n}}}$$

$$= 1$$

Therefore $\sum_{n=2}^{\infty}\frac{5+n}{n\sqrt{n+1}}$ diverges by the limit comparison test.

* 7.) (10 pts) Does $\sum_{n=1}^{\infty}\frac{(-1)^n}{1+n^2}$ converge or diverge? If it converges, is it conditional or absolute convergence? Justify your answer.

show abs. convergence by proving $\sum_{n=1}^{\infty}\left|\frac{(-1)^n}{1+n^2}\right| = \sum_{n=1}^{\infty}\frac{1}{1+n^2}$ converges.

$\sum_{n=1}^{\infty}\frac{1}{1+n^2} \le \sum_{n=1}^{\infty}\frac{1}{n^2}$ which is a convergent p-series.

Therefore $\sum_{n=1}^{\infty}\frac{(-1)^n}{1+n^2}$ is absolutely convergent.

* 8.) (10 pts) Does $\sum_{n=1}^{\infty}\frac{(-1)^n 7^n}{n!\sqrt{n+1}}$ diverge? If not, is it conditionally or absolutely convergent? Justify your answer.

Ratio Test.

$$\lim_{n\to\infty}\left|\frac{\frac{(-1)^{n+1}7^{n+1}}{(n+1)!\sqrt{n+2}}}{\frac{(-1)^n 7^n}{n!\sqrt{n+1}}}\right| = \lim_{n\to\infty}\frac{7\sqrt{n+1}}{(n+1)\sqrt{n+2}}$$

$$= 0 < 1$$

Therefore the series diverges by the ratio test.