50's	60's	70's	80's	90's	100 %
3	8	9	5	4	0

Test 1 (Version φ)
Dusty Wilson
Math 153

high: 97.3%
mean: 73.8%
median: 71.6%

No work = no credit

No Symbolic Calculators

Name: Key 7:14

I myself, a professional mathematician, on re-reading my own work find it strains my mental powers to recall to mind from the figures the meanings of the demonstrations, meanings which I myself originally put into the figures and the text from my mind. But when I attempt to remedy the obscurity of the material by putting in extra words, I see myself falling into the opposite fault of becoming chatty in something mathematical.

Johannes Kepler (1597 - 1630)
German astronomer

Warm-ups (1 pt each): $\vec{i}\times\vec{j}=$ $\vec{k}$ $\quad \vec{j}-\vec{i}=$ $\langle -1,1,0\rangle$ $\quad \vec{j}\cdot\vec{k}=$ 0

1.) (1 pt) Based upon the quote above, how easily did Kepler understand his earlier work? Answer using complete English sentences.

It was hard to read his own work.

2.) (10 pts) Consider $\vec{u}=\langle 1,5,3\rangle$ and $\vec{v}=\langle 2,5,4\rangle$.

a.) Find $\vec{u}\cdot\vec{v}$

$\langle 1,5,3\rangle\cdot\langle 2,5,4\rangle$

$= 2+25+12$

$= 39$

b.) Find the angle between $\vec{u}$ and $\vec{v}$.

$\theta = \cos^{-1}\left(\frac{39}{\sqrt{35}\cdot\sqrt{45}}\right)$

$\approx 10.7^0$

c.) Find $\vec{u}\times\vec{v}$

$$\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 5 & 3 \\ 2 & 5 & 4 \end{vmatrix}$$

$= \langle 5,2,-5\rangle$

d.) Find $\text{comp}_{\vec{u}}\vec{v}$

$\text{comp}_{\vec{u}}\vec{v} = \frac{39}{\sqrt{35}}$

e.) Find $\text{proj}_{\vec{u}}\vec{v}$

$\text{proj}_{\vec{u}}\vec{v} = \frac{39}{35}\langle 1,5,3\rangle$

3.) (10 pts) Where does the line through $A(2,0,1)$ and $B(0,3,2)$ intersect the plane $x+2y-z=7$

The line: $\vec{r}(t) = \langle 2,0,1\rangle + t\langle -2,3,1\rangle$

$= \langle 2-2t,\ 3t,\ t+1\rangle$

sub into the line.

$(2-2t) + 2(3t) - (t+1) = 7$

$\Rightarrow 3t+1 = 7$

$\Rightarrow 3t = 6$

$\Rightarrow t = 2$

Find the point.

$\vec{r}(2) = (-2,6,3)$

4.) (10 pts) Find an equation of the plane that includes the lines $x = \dfrac{y-9}{8} = \dfrac{z-1}{4}$ and $\vec{r}(t) = (1-t)\langle 1,2,4\rangle + t\langle 1,4,6\rangle$ $= \langle 1,\ 2+2t,\ 4+2t\rangle$

Vectors "on" the plane.

$\langle 1,8,4\rangle$

and $\langle 0,2,2\rangle$

w/ normal $\langle 8,-2,2\rangle$

The plane: $8(x-1) - 2(y-2) + 2(z-4) = 0$

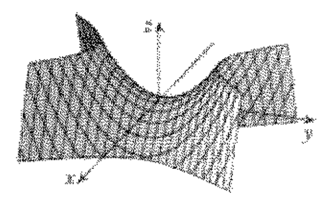
Skew lines are not coplanar.

5.) (5 pts) Write the equation of the sphere with radius 3, centered at the point (0,1,2).

$$x^2 + (y-1)^2 + (z-2)^2 = 9$$

6.) (5 pts) Write a possible equation for the quadric surface pictured below.

$$z = y^2 - x^2$$



7.) (10 pts) Find the equation of the tangent line to the polar curve $r = 4\sin\theta$ when $\theta = \frac{\pi}{3}$

slope

$$y = r\sin\theta \qquad x = r\cos\theta \quad \Rightarrow \quad \frac{dy}{dx} = \frac{r's + rc}{r'c - rs}\Bigg|_{\theta=\frac{\pi}{3}} \quad \frac{\cancel{2}\frac{\sqrt{3}}{\cancel{2}} + \cancel{2}\sqrt{3}\cdot\frac{1}{\cancel{2}}}{\cancel{2}\cdot\frac{1}{\cancel{2}} - \cancel{2}\sqrt{3}\,\frac{\sqrt{3}}{\cancel{2}}} = \frac{2\sqrt{3}}{-2} = -\sqrt{3}$$

$$r\left(\tfrac{\pi}{3}\right) = 4\cdot\frac{\sqrt{3}}{2} = 2\sqrt{3}$$

$$r'\left(\tfrac{\pi}{3}\right) = 4\cdot\frac{1}{2} = 2$$

point

$$x = 2\sqrt{3}\left(\tfrac{1}{2}\right) = \sqrt{3}$$

$$y = \cancel{2}\sqrt{3}\cdot\frac{\sqrt{3}}{\cancel{2}} = 3$$

line:

$$y - 3 = -\sqrt{3}\,(x - \sqrt{3})$$

7:rb

7:36

8.) (10 pts) Use techniques developed in this course to verify that the area of a circle with radius 6 is 36π.

Hint: Begin by writing an equation for a circle of radius 6 centered at the origin.

$r = 6$

$$A = \int_0^{2\pi} \frac{1}{2} \cdot 6^2 \, d\theta$$

$$= \int_0^{2\pi} 18 \, d\theta$$

$$= 36\pi$$

polar soln.

$\vec{r}(t) = \langle 6\cos t, 6\sin t \rangle$

$$A = 2\int_0^{\pi} 6\sin t \cdot (-6\sin t)\, dt$$

$$= 72\int_0^{\pi} \sin^2 t \, dt$$

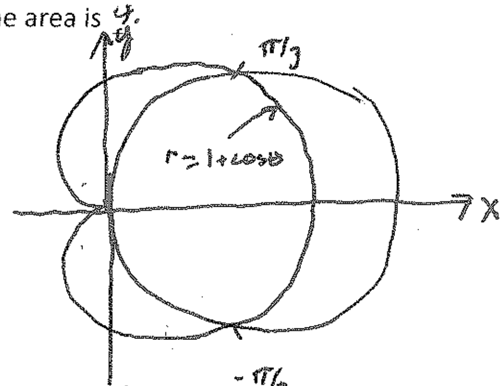
$$= 72\int_0^{\pi} \frac{1-\cos 2\theta}{2}\, dt$$

$$= 36\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi} \quad 36\pi$$

parametric soln.

9.) (10 pts) Set up an integral to find the arclength of the part of the cardioid $r = 1 + \cos\theta$ that lies inside $r = 3\cos\theta$.

Note: You may evaluate the integral to verify the area is 4.

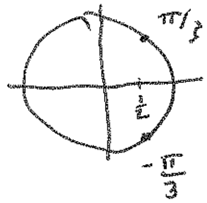


Find intersection points.

solve $3\cos\theta = 1 + \cos\theta$

$\Rightarrow 2\cos\theta = 1$

$\Rightarrow \cos\theta = \frac{1}{2}$



$$L = 2\int_0^{\pi/3} \sqrt{(1+\cos\theta)^2 + (-\sin\theta)^2}\, d\theta$$

$$= 2\int_0^{\pi/3} \sqrt{1 + 2\cos\theta + \cos^2\theta + \sin^2\theta}\, d\theta$$

$$= 2\sqrt{2}\int_0^{\pi/3} \sqrt{1+\cos\theta}\, d\theta$$

$$= 4$$

7:40

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Dusty Wilson
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Johannes Kepler (1597 - 1630)
German astronomer

Warm-ups (1 pt each): $\vec{j} \times \vec{k} = \vec{i}$ $\quad \vec{i} - \vec{j} = \langle 1, -1, 0 \rangle$ $\quad \vec{k} \cdot \vec{k} = 1$

1.) (1 pt) Based upon the quote above, how easily did Kepler understand his earlier work? Answer using complete English sentences.

He struggled w/ his own work.

2.) (10 pts) Consider $\vec{u} = \langle 1,5,2 \rangle$ and $\vec{v} = \langle 2,3,0 \rangle$.

a.) Find $\vec{u} \cdot \vec{v}$

$= \langle 1,5,2 \rangle \cdot \langle 2,3,0 \rangle$

$= 2 + 15 + 0$

$= 17$

b.) Find the angle between $\vec{u}$ and $\vec{v}$ in degrees (to 1 decimal place).

$\theta = \cos^{-1}\left(\frac{17}{\sqrt{30}\sqrt{13}}\right)$

$= 30.6°$

c.) Find $\vec{u} \times \vec{v}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 5 & 2 \\ 2 & 3 & 0 \end{vmatrix}$$

$= \langle -6, 4, -7 \rangle$

d.) Find $\text{comp}_{\vec{u}}\vec{v} = \frac{17}{\sqrt{30}}$

e.) Find $\text{proj}_{\vec{u}}\vec{v} = \frac{17}{30}\langle 1,5,2 \rangle$

3.) (10 pts) Find an equation of the plane that includes the lines $\frac{x-1}{4}=\frac{y-2}{9}=z$ and $\vec{r}(t)=(1-t)\langle 2,2,0\rangle+t\langle 1,8,1\rangle$ $= \langle 2-2t, 2-2t, 0\rangle + \langle t, 8t, t\rangle = \langle 2-t, 2+6t, t\rangle$

Find vectors in the direction of each line.

$\vec{u} = \langle 4, 9, 1\rangle$

$\vec{v} = \langle -1, 6, 1\rangle$

Skew lines are not coplanar.

w/ the Normal $\vec{u}\times\vec{v} = \langle 3, -5, 33\rangle$

so the plane is: $3(x-1) + -5(y-2) + 33(z-0) = 0$

$$3x - 5y + 33z = -7$$

check:

line 1 $\vec{r}_1(t) = \langle 4t+1, 9t+2, t\rangle$

$\to 3(4t+1) - 5(9t+2) + 33t = -7$

$12t + 3 - 45t - 10 + 33t = -7$ ✓

line 2 $\vec{r}_2(t) = \langle 2-t, 2+6t, t\rangle$

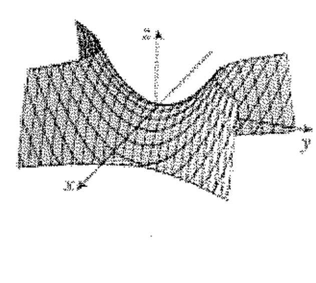
$\to 3(2-t) - 5(2+6t) + 33t = -7$ ✗

4.) (5 pts) Write the equation of the sphere with radius 4, centered at the point (2,1,0).

$$(x-2)^2 + (y-1)^2 + (z-0)^2 = 16$$

5.) (5 pts) Write a possible equation for the quadric surface pictured below.

$$z = y^2 - x^2$$



6.) (10 pts) Where does the line through $A(-1,1,3)$ and $B(0,3,2)$ intersect the plane $3x-y+2z=5$?

Find the line thru A & B.

$\vec{r}(t) = (1-t)\langle -1,1,3\rangle + t\langle 0,3,2\rangle$

$= \langle -1+t, 1+2t, 3-t\rangle$

Substitute into the plane eqt. to find t.

solve $3(-1+t) - 1(1+2t) + 2(3-t) = 5$

$\Rightarrow -3+3t-1-2t+6-2t = 5$

$\Rightarrow \cancel{8}^{2} - t = 5$

$\Rightarrow t = -3$

Find the point

$\vec{r}(3) = \langle -4, -5, 6\rangle$

7.) (10 pts) Use techniques developed in this course to verify that the area of a circle with radius 7 is 49π.

Hint: Begin by writing an equation for a circle of radius 7 centered at the origin.

polar

$r = 7$

$\Rightarrow \text{Area} = \int_0^{2\pi} \frac{1}{2}\cdot 7^2\, d\theta$

$= \frac{49}{2}\cdot 2\pi$

$= 49\pi.$

parametric

$\vec{r}(t) = \langle 7\cos t, 7\sin t\rangle$

orientation

$\Rightarrow \text{Area} = -2\int_0^{\pi} 7\sin t\,(-7\sin t)\,dt$

$= 98\int_0^{\pi} \sin^2 t\, dt$

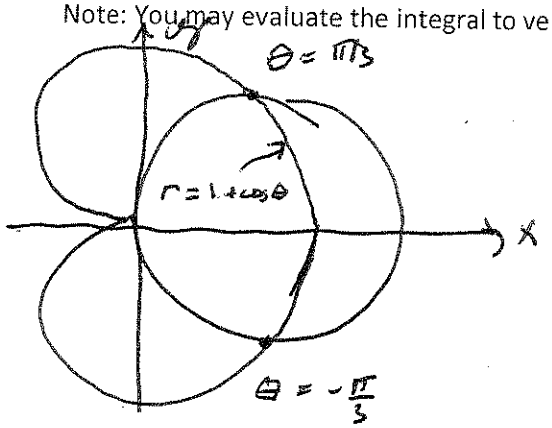
$= 49\int_0^{\pi} 1-\cos 2t\, dt$

$= 49\left[t - \frac{\sin 2t}{2}\right]_0^{\pi}$

$= 49\pi$

8.) (10 pts) Set up an integral to find the arclength of the part of the cardioid $r = 1+\cos\theta$ that lies inside $r = 3\cos\theta$.

Note: You may evaluate the integral to verify the area is 4.



$$L = 2\int_0^{\pi/3} \sqrt{(1+\cos\theta)^2 + (-\sin\theta)^2}\, d\theta$$

$$= 2\int_0^{\pi/3} \sqrt{2+2\cos\theta}\, d\theta$$

$$= 2\sqrt{2}\int_0^{\pi/3} \sqrt{2}\cos\frac{\theta}{2}\, d\theta \quad \text{Half-angle identity}$$

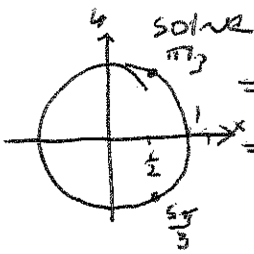
$$= 4\left[2\sin\frac{\theta}{2}\right]_0^{\pi/3}$$

$$= 8\left(\frac{1}{2}-0\right)$$

$$= 4.$$

Find the intersection point

solve $1+\cos\theta = 3\cos\theta$



$\Rightarrow 1 = 2\cos\theta$

$\Rightarrow \frac{1}{2} = \cos\theta$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

9.) (10 pts) Use the methods developed in this course to find the equation of the tangent line to the polar curve $r = 4\cos\theta$ when $\theta = \frac{\pi}{3}$

Find the point.

$x = r\cos\theta$

$= 4\cos^2\theta \Big|_{\theta=\frac{\pi}{3}} \quad 1$

$y = r\sin\theta$

$= 4\sin\theta\cos\theta \Big|_{\theta=\frac{\pi}{3}} \quad \sqrt{3}$

point $(1, \sqrt{3})$

Find the slope

$$\frac{dy}{dx} = \frac{r's + rc}{r'c - rs}$$

At $\theta = \frac{\pi}{3}$

$r = 2,\ r' = -2\sqrt{3},\ s = \frac{\sqrt{3}}{2},\ c = \frac{1}{2}$

$$\Rightarrow \frac{dy}{dx} = \frac{-\cancel{2}\sqrt{3}\cdot\frac{\sqrt{3}}{\cancel{2}} + 2\cdot\frac{1}{2}}{-\cancel{2}\sqrt{3}\cdot\frac{1}{\cancel{2}} - \cancel{2}\left(\frac{\sqrt{3}}{\cancel{2}}\right)}$$

$$= \frac{-3+1}{-2\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}}$$

Line: $y - \sqrt{3} = \frac{1}{\sqrt{3}}(x-1)$