Group Quiz 7
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Math 153

Name: key

No work = no credit

1.) Evaluate the indefinite integral $\int \frac{\arctan(3x)}{x}\,dx$ as a power series and find its interval and radius of convergence.

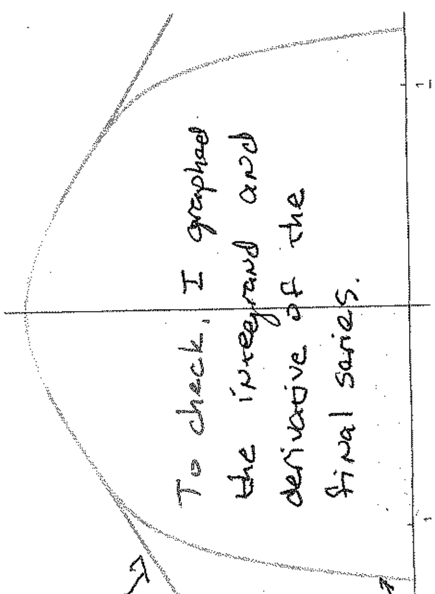
← This is not defined @ $x=0$. However, the improper integral is defined when evaluated as a limit.

$$\frac{d}{dx}\arctan(3x) = \frac{3}{1+9x^2} = \frac{3}{1-(-9x^2)} = \sum_{n=0}^{\infty} 3(-9x^2)^n = \sum_{n=0}^{\infty} (-1)^n 3^{2n+1} x^{2n}$$

$$\Rightarrow \arctan(3x) = \sum_{n=0}^{\infty} \int (-1)^n 3^{2n+1} x^{2n}\,dx = C + \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1} x^{2n+1}}{2n+1}$$

$C = 0$ since $\tan^{-1}(0) = 0$

$$\Rightarrow \frac{\arctan(3x)}{x} = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1} x^{2n}}{2n+1}$$



f(x)

$f[x\_] = \frac{\text{ArcTan}[3x]}{x};$

$g[x\_] = \sum_{n=0}^{5} D\left[\frac{(-1)^n 3^{2n+1} x^{2n+1}}{(2n+1)^2}, x\right];$

g(x)

$$\int \frac{\arctan 3x}{x}\,dx = \sum_{n=0}^{\infty} \int \frac{(-1)^n 3^{2n+1} x^{2n}}{2n+1}\,dx = C + \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1} x^{2n+1}}{(2n+1)^2}$$

Find the IOC and ROC

Ratio Test

$$\lim_{n\to\infty} \left| \frac{\frac{(-1)^{n+1} 3^{2(n+1)+1} x^{2(n+1)+1}}{[2(n+1)+1]^2}}{\frac{(-1)^n 3^{2n+1} x^{2n+1}}{(2n+1)^2}} \right| = \lim_{n\to\infty} \left| \frac{9x^2(2n+1)^2}{(2n+3)^2} \right| = |9x^2| < 1$$

when $-\frac{1}{3} < x < \frac{1}{3}$

check endpoints: both conv. abs by using the LCT and comparing $\sum \frac{1}{(2n+1)^2}$ to $\sum \frac{1}{n^2}$ which is a convergent ...

I.O.C. $-\frac{1}{3} \le x \le \frac{1}{3}$

R.O.C. $R = \frac{1}{3}$

2.) Find the Taylor Series for $f(x)=\sqrt{x}$ centered at $a=25$ and find its ~~interval~~ radius of convergence. Approximate $\sqrt{26}$ with $T_3(26)$. Σ Notation is not required.

n	$f^{(n)}(x)$	$f^{(n)}(25)$
0	$x^{1/2}$	5
1	$\frac{1}{2}x^{-1/2}$	$\frac{1}{2}\cdot\frac{1}{5}$
2	$\frac{1}{2}\cdot\frac{-1}{2}x^{-3/2}$	$-\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{5^3}$
3	$\frac{3}{2^3}x^{-5/2}$	$\frac{3}{2^3}\cdot\frac{1}{5^5}$
4	$-\frac{3(5)}{2^4}x^{-7/2}$	$-\frac{3(5)}{2^4}\cdot\frac{1}{5^7}$

Find the I.o.C. w/ the Ratio Test.

$$\lim_{n\to\infty}\left|\frac{\dfrac{(-1)^{n+1}\,1\cdot3\cdot5\cdot7\cdots(2n+1)\,(x-25)^{n+1}}{2^{n+1}\,5^{2(n+1)+1}\cdot(n+1)!}}{\dfrac{(-1)^n\,1\cdot3\cdot5\cdot7\cdots(2n-1)\,(x-25)^n}{2^n\,5^{2n+1}\cdot n!}}\right|$$

$$=\lim_{n\to\infty}\left|\frac{(2n+1)(x-25)}{2\cdot25\,(n+1)}\right|<1$$

$$\Rightarrow \left|\frac{x-25}{25}\right|<1 \Rightarrow \text{I.O.C. } 0\le x\le 50$$

↑ end pts work

$R=25$

$$f(x)=\sqrt{x}=5+\frac{(x-25)}{2\cdot5\cdot1!}-\frac{1(x-25)^2}{2^2\cdot5^3\cdot2!}+\frac{3(x-25)^3}{2^3\cdot5^5\cdot3!}-\frac{3\cdot5(x-25)^4}{2^4\cdot5^7\cdot4!}+\cdots$$

|← $T_3(x)$ →|

$T_3(26)=5.09902$

3.) Use the error bounding methods introduced in this course to determine how many terms of the Maclaurin series for $\ln(1+x)$ are needed to estimate $\ln(1.5)$ to within 0.0005?

$$\ln|1+x|=\int\frac{dx}{1+x}=\sum_{n=0}^{\infty}\int(-1)^n x^n\,dx$$

$$=C+\sum_{n=0}^{\infty}\frac{(-1)^n x^{n+1}}{n+1}$$

↑ $C=0$ since $\ln(1)=0$

$$\Rightarrow \ln(1+x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}x^n}{n}$$

and $$\ln(1.5)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\cdot1}{2^n\cdot n}$$

Bound w/ the Alternating Series Estimating series.

Solve: $$\frac{1}{2^{n+1}\cdot(n+1)}\le 0.0005$$

$$\Rightarrow 2000\le 2^{n+1}(n+1)$$

$$\Rightarrow 7\le n$$

so $$\ln(1.5)\approx\sum_{n=1}^{7}\frac{(-1)^{n+1}}{2^n\cdot n}$$

seven terms are needed

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Abs[Log[1.5] - Sum[(-1)^(n+1)/(2^n n), {n,1,7}] // N]
0.000338463
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