Group Quiz 6
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Math 153

Name: Key

No work = no credit

1.) Determine whether the series $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}\sqrt{n^4+16}}{3n^4-5}$ is convergent or divergent. If it converges, is it conditionally or absolutely convergent? Justify your results.

Show Abs convergence w/ the LCT.

$$\text{LCT on } \sum_{n=2}^{\infty} \left| \frac{(-1)^{n+1}\sqrt{n^4+16}}{3n^4-5} \right| = \sum_{n=2}^{\infty} \frac{\sqrt{n^4+16}}{3n^4-5}$$

compare to $\sum_{n=2}^{\infty} \frac{1}{n^2}$

1 pt.

$$\lim_{n\to\infty} \frac{\frac{\sqrt{n^4+16}}{3n^4-5}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{\sqrt{n^4+16}}{3n^4-5} \cdot \frac{n^2}{1}$$

$$= \lim_{n\to\infty} \frac{\sqrt{1+16/n^4} \cdot \frac{n^2}{n^2}}{3 - \frac{5}{n^4}}$$

$$= \frac{1}{3} \longleftarrow \text{a non-zero constant.}$$

Since $\sum_{n=2}^{\infty} \frac{1}{n^2}$ converges absolutely, so does the original series by the limit comparison test.

2.) Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 3 \cdot n^3}{(n+4) \cdot 5^{n+2}}$ is convergent or divergent. If it converges, is it conditionally or absolutely convergent? Justify your results.

ratio test

$$\lim_{n \to \infty} \left| \frac{(-1)^{n+2} \cancel{3} (n+1)^3}{(n+5)\, 5^{n+3}} \cdot \frac{(n+4) \cdot 5^{n+2}}{(-1)^{n+1} \cancel{3} \cdot n^3} \right|$$

$$= \lim_{n \to \infty} \frac{(n+1)^3 (n+4)}{5 n^3 (n+5)}$$

$$= \frac{1}{5} < 1$$

Hence the series converges absolutely by the ratio test.

3.) Determine whether the series $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$ is convergent or divergent. If it converges, is it conditionally or absolutely convergent? Justify your results.

limit comparison test w/ $\sum_{n=1}^{\infty} \frac{1}{n^2}$

$$\lim_{n \to \infty} \frac{\sin\left(\frac{1}{n^2}\right)}{\frac{1}{n^2}} = \lim_{x \to \infty} \frac{\sin\left(\frac{1}{x^2}\right)}{\frac{1}{x^2}}$$ x is a continuous variable

$$\overset{H}{=} \lim_{x \to \infty} \frac{\cos\left(\frac{1}{x^2}\right) \cdot \frac{-2}{x^3}}{\frac{-2}{x^3}}$$

$= 1 \leftarrow$ a non-zero constant.

Since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is a convergent p-series,

our original series converges absolutely by the L.C.T.