Group Quiz 5
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Math 153

No work = no credit

Name: key

1.) Determine whether the series $\displaystyle\sum_{n=1}^{\infty}\left[\frac{5\cdot 2^{3n-1}}{3^{2n}}-\ln\left(\frac{n+3}{n}\right)+\frac{3n}{16n^4+1}\right]$ is convergent or divergent.

only allowed → if all the series converge.

$$= \underbrace{\sum_{n=1}^{\infty}\frac{5}{2}\left(\frac{8}{9}\right)^n}_{20} - \underbrace{\sum_{n=1}^{\infty}\left[\ln(n+3)-\ln n\right]}_{\text{Diverges.}} + \underbrace{\sum_{n=1}^{\infty}\frac{3n}{16n^4+1}}_{\text{converges.}}$$

(1), (2), (3)

Since (2) diverges, we cannot rewrite the series of a sum as the sum of series (at least w/o justification). The next page proves divergence.

(1) Geometric series

$a = \frac{40}{18}$ and $r = \frac{8}{9}$

$$\text{sum} = \frac{\frac{40}{18}}{1-\frac{8}{9}} = \frac{40}{18}\cdot\frac{9}{1} = 20$$

(2) telescoping series.

$$s_k = (\ln 4 - \ln 1) + (\ln 5 - \ln 2) + (\ln 6 - \ln 3) + (\ln 7 - \ln 4) + (\ln 8 - \ln 5) + \dots + (\ln(k+1) - \ln(k-2)) + (\ln(k+2) - \ln(k-1)) + (\ln(k+3) - \ln k)$$

$$= \ln(k+3) + \ln(k+2) + \ln(k+1) - \ln 3 - \ln 2$$

And $\lim_{k\to\infty} s_k = \infty$ so the series diverges.

(3) L.C.T.

(a) $\displaystyle\lim_{n\to\infty}\frac{\frac{3n}{16n^4+1}}{\frac{1}{n^3}} = \frac{3}{16}$

(b) Since $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^3}$ is a convergent p-series, $\displaystyle\sum_{n=1}^{\infty}\frac{3n}{16n^4+1}$ converges by the L.C.T.

$$\sum_{n=1}^{\infty}\left(\frac{20}{9}\left(\frac{8}{9}\right)^{n-1}-\ln\left(\frac{n+3}{n}\right)+\frac{3n}{16n^4+1}\right)$$

This cannot be split into the sum/difference of three series as $\sum \ln\left(\frac{n+3}{n}\right)$ diverges.

We will use the more primitive approach and consider the limit of the partial sums s_k.

$$s_k=\sum_{n=1}^{k}\left(\frac{20}{9}\left(\frac{8}{9}\right)^{n-1}-\ln\left(\frac{n+3}{n}\right)+\frac{3n}{16n^4+1}\right)$$

$$=\underbrace{\frac{20}{9}\,\frac{\left(1-\left(\frac{8}{9}\right)^k\right)}{1-\frac{8}{9}}}_{<\,20 \text{ for positive } k}-\ln(k+3)-\ln(k+2)-\ln(k+1)+\ln 3+\ln 2+\underbrace{\sum_{n=1}^{k}\frac{3n}{16n^4+1}}_{<\,\frac{3}{16}\sum_{n=1}^{k}\frac{1}{n^3}}$$

$$<c_1-\ln\left[(k+1)(k+2)(k+3)\right]+\frac{3}{16}\sum_{n=1}^{k}\frac{1}{n^3}$$

$$<c_1-\ln\left[(k+1)(k+2)(k+3)\right]+\frac{3}{16}\sum_{n=1}^{\infty}\frac{1}{n^3}$$

$$<c_1-\ln\left[(k+1)(k+2)(k+3)\right]+\frac{3}{16}\left(1+\underbrace{\int_1^{\infty}\frac{dx}{x^3}}_{1/2}\right)$$

$$=c_2-\ln\left[(k+1)(k+2)(k+3)\right]$$

$$\Rightarrow \lim_{k\to\infty}s_k<\lim_{k\to\infty}\left[c_2-\ln\left[(k+1)(k+2)(k+3)\right]\right]=-\infty$$

$\Rightarrow \lim_{k\to\infty}s_k=-\infty$ and the series diverges.

2.) Find the values of x for which the series $\sum_{n=1}^{\infty}(-2)^n\,[\ln x]^n$ converges and find the sum of the series for those values of x.

$$\sum_{n=1}^{\infty}(-2)^n[\ln x]^n = \sum_{n=1}^{\infty}[-2\ln x]^n = \frac{-2\ln x}{1+2\ln x}$$

geometric series

$a = -2\ln x$

$r = -2\ln x$

when $-1 < -2\ln x < 1$

$\Rightarrow \frac{1}{2} > \ln x > -\frac{1}{2}$

$\Rightarrow \sqrt{e} > x > \frac{1}{\sqrt{e}}$

3.) Cool things happen in infinite series. For example, $s=\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$?!?! Use the methods developed in this course to find a value of n that will ensure that the error in the approximation $s \approx s_n$ is less than 0.00005.

$$\int_{n+1}^{\infty}\frac{dx}{x^2} \le R_n \le \int_{n}^{\infty}\frac{dx}{x^2} \le 0.00005$$

$\Rightarrow$ we must solve $\lim_{t\to\infty}\int_n^t \frac{dx}{x^2} \le 0.00005$

$$\Rightarrow \lim_{t\to\infty}\left[-x^{-1}\right]_n^t \quad \lim_{t\to\infty}\left(\frac{1}{n} - \cancel{\frac{1}{t}}^{\,\to 0}\right) \le 0.00005$$

$$\Rightarrow \frac{1}{n} \le 0.00005$$

$$\Rightarrow \frac{1}{0.00005} \le n$$

n must be at least 20,000.

In[10]:= $\frac{\pi^2}{6} - \sum_{n=1}^{100} \frac{1}{n^2}$ // N

Out[10]= 0.00995017

In[11]:= $\frac{\pi^2}{6} - \sum_{n=1}^{1000} \frac{1}{n^2}$ // N

Out[11]= 0.0009995

In[12]:= $\frac{\pi^2}{6} - \sum_{n=1}^{10\,000} \frac{1}{n^2}$ // N

Out[12]= 0.000099995

In[14]:= $\frac{\pi^2}{6} - \sum_{n=1}^{19\,999} \frac{1}{n^2}$ // N

Out[14]= 0.0000500013

$\frac{\pi^2}{6} - \sum_{n=1}^{20\,000} \frac{1}{n^2}$ // N

Out[13]= 0.0000499988