Group Quiz 4 Name: Key
Dusty Wilson
Math 153

No work = no credit

1.) Given $\bar{r}(t) = \langle e^{2t}, 1+e^{2t}\cos(t), 3+e^{2t}\sin(t)\rangle$, find the unit tangent, unit normal, and binormal vectors at $(1, 2, 3)$. By observation, $t=0$.

$$\vec{r}\,'(t) = \langle 2e^{2t}, 2e^{2t}\cos t - e^{2t}\sin t, 2e^{2t}\sin t + e^{2t}\cos t\rangle$$

$$= e^{2t}\langle 2, 2\cos t - \sin t, 2\sin t + \cos t\rangle$$

$$\Rightarrow |\vec{r}\,'(t)| = e^{2t}\sqrt{4 + 4c^2 - 4\cancel{sc} + s^2 + 4s^2 + 4\cancel{sc} + c^2}$$

$$= 3e^{2t}$$

$$\Rightarrow \vec{T}(t) = \frac{1}{3}\langle 2, 2\cos t - \sin t, 2\sin t + \cos t\rangle$$

and $\vec{T}(0) = \frac{1}{3}\langle 2, 2, 1\rangle = \langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3}\rangle$

$$\Rightarrow \vec{T}\,'(t) = \frac{1}{3}\langle 0, -2\sin t - \cos t, 2\cos t - \sin t\rangle$$

and $\vec{T}\,'(0) = \langle 0, -\frac{1}{3}, \frac{2}{3}\rangle$

$$\Rightarrow |\vec{T}\,'(0)| = \sqrt{0 + \frac{1}{9} + \frac{4}{9}} = \frac{\sqrt{5}}{3}$$

and $\vec{N}(0) = \frac{3}{\sqrt{5}}\langle 0, -\frac{1}{3}, \frac{2}{3}\rangle = \langle 0, \frac{-1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\rangle$

$$\Rightarrow \vec{B}(0) = \langle \frac{5}{3\sqrt{5}}, \frac{-4}{3\sqrt{5}}, \frac{-2}{3\sqrt{5}}\rangle$$

2.) For the same position function $\vec{r}(t)=\langle e^{2t},\, 1+e^{2t}\cos(t),\, 3+e^{2t}\sin(t)\rangle$, (A.) write the acceleration in terms of its tangential and normal components of acceleration at $(1, 2, 3)$. (B.) Find the angle between the acceleration and velocity vectors. (C.) Find the angle between the unit tangent and unit normal vectors (i.e., the tangential and normal projections of the acceleration vector).

$$\vec{r}\,'(t) = e^{2t}\langle 2,\, 2c-s,\, 2s+c\rangle\Big|_{t=0} \quad \langle 2,2,1\rangle \text{ w/ magnitude } 3$$

$$\vec{r}\,''(t) = 2e^{2t}\langle 2,\, 2c-s,\, 2s+c\rangle + e^{2t}\langle 0,\, -2s-c,\, 2c-s\rangle$$

$$= e^{2t}\langle 4,\, 3c-4s,\, 3s+4c\rangle\Big|_{t=0} \quad \langle 4,3,4\rangle$$

$$a_T = \frac{\vec{r}\,'\cdot\vec{r}\,''}{|\vec{r}\,'|} = \frac{18}{3}$$

$$a_N = \frac{|\vec{r}\,'\times\vec{r}\,''|}{|\vec{r}\,'|} = \frac{|\langle 5,-4,-2\rangle|}{3} = \frac{\sqrt{45}}{3} = \sqrt{5}$$

$$\boxed{\begin{array}{l}(B.)\ \cos^{-1}\left(\frac{18}{3\sqrt{41}}\right) = 20.4^\circ \\ (C.)\ \vec{T}\perp\vec{N} \text{ so the angle between is } 90^\circ\end{array}}$$

(A.) $\vec{a}(0) = \vec{r}\,''(0) = \langle 4,3,4\rangle = \frac{18}{3}\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3}\rangle + \sqrt{5}\langle 0, -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\rangle$

3.) Given the same function $\vec{r}(t)=\langle e^{2t},\, 1+e^{2t}\cos(t),\, 3+e^{2t}\sin(t)\rangle$, find the equation of the osculating plane and kissing circle of the curve at $(1, 2, 3)$.

Note: This involves a lot of algebra and calculus. To help you know you are on the right track, the equation of the osculating plane is: $\frac{\sqrt{5}}{3}(x-1)-\frac{4}{3\sqrt{5}}(y-2)-\frac{2}{3\sqrt{5}}(z-3)=0$ and the curvature is about 0.25.

we need the curvature: $k = \frac{a_N}{|\vec{r}\,'|^2} = \frac{\sqrt{5}}{9}$

plane: $\frac{5}{3\sqrt{5}}(x-1) - \frac{4}{3\sqrt{5}}(y-2) - \frac{2}{3\sqrt{5}}(z-3) = 0$

kissing circle $\vec{r}(\theta) = \langle 1,2,3\rangle + \frac{9}{\sqrt{5}}\langle 0, -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\rangle +$

$$\frac{9}{\sqrt{5}}\left(\cos\theta\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3}\rangle + \sin\theta\langle 0, -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\rangle\right)$$

$$= \langle 1, -\frac{1}{5}, \frac{33}{5}\rangle + \frac{3}{\sqrt{5}}\cos\theta\langle 2,2,1\rangle + \frac{9}{5}\sin\theta\langle 0,-1,2\rangle$$