Group Quiz 2
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Math 153

Name: Key.

No work = no credit

1.) Basic lines and planes

a.) Find the line through the points A(3,2,1) and B(4,5,6). What are the coordinates of at least one other point on the line?

Any form is okay.

$\overrightarrow{AB} = \langle 1, 3, 5 \rangle$

line: $\langle x, y, z \rangle = \langle 3, 2, 1 \rangle + t\langle 1, 3, 5 \rangle$

or $x = 3 + t$; $y = 2 + 3t$; $z = 1 + 5t$

or $x - 3 = \dfrac{y-2}{3} = \dfrac{z-1}{5}$

or $\vec{r}(t) = (1-t)\langle 3, 2, 1 \rangle + t\langle 4, 5, 6 \rangle$

or ...

One other point

$t = 2$: $(5, 8, 11)$

b.) Find the plane through the points A(2,4,6), B(0,1,3), and C(1,4,1). What are the coordinates of at least one other point on the plane?

$\overrightarrow{AB} = \langle -2, -3, -3 \rangle$

$\overrightarrow{AC} = \langle -1, 0, -5 \rangle$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & -3 & -3 \\ -1 & 0 & -5 \end{vmatrix}$$

$= \langle 15, -7, -3 \rangle$

plane: $15(x-2) - 7(y-4) - 3(z-6) = 0$

$\Rightarrow 15x - 30 - 7y + 28 - 3z + 18 = 0$

$\Rightarrow 15x - 7y - 3z = -16$

one other point

if $x = 0$

$y = 0$

then $0 + 0 - 3z = -16$

$\Rightarrow z = \dfrac{16}{3}$

point $(0, 0, \frac{16}{3})$

2.) Find another plane that shares the same line of intersection as the planes $x+y+z=1$ and $x+y=2$

<u>soln 1:</u> subtract.

$$\begin{aligned} x+y+z&=1 \\ x+y\quad&=2 \\ \hline z&=-1 \end{aligned}$$

since the line of intersection lies on the plane $z=-1$, then $z=-1$ is a soln.

<u>soln 2:</u> Find the line of intersection

$y=-x+2\ ,\ z=-1$

$\Rightarrow \langle x,y,z\rangle = \langle t, -t+2, -1\rangle$ 7/15 is this is clear.

we just need a plane. So pic an arbitrary point that isn't on either plane. The origin would be fast but I'll choose $A(1,2,3)$ so you can see what is happening.

line: $\langle 1,-1,0\rangle$

line to A: $\langle 1,2,3\rangle - \langle 0,2,-1\rangle = \langle 1,0,4\rangle$ } vectors of the new plane.

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 0 \\ 1 & 0 & 4 \end{vmatrix} = \langle -4,-4,1\rangle$$ is the normal.

A plane that satisfies the condition is:

$\boxed{-4(x-1)-4(y-2)+1(z-3)=0}$

3.) Find exact the distance from the plane $4x+5y+6z=7$ the point where the lines $L_1:\langle x,y,z\rangle=\langle 2t+1,3t+2,4t+3\rangle$ and $L_2: x-2=\dfrac{y-4}{2}=\dfrac{z+1}{-4}$ intersect.

① Find the point of intersection.

$x:\ 2t+1=s+2 \quad\Rightarrow\quad 2t-s=1 \quad\Rightarrow\quad t=0$

$y:\ 3t+2=2s+4 \quad\quad 3t-2s=2 \quad\quad s=-1$

check z on L_1 is 3 and z on L_2 is $-4s-1=3$ ✓

so the point is $(1,2,3)$

② Find the distance.

$$D=\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}=\frac{|4(1)+5(2)+6(3)-7|}{\sqrt{4^2+5^2+6^2}}=\frac{25}{\sqrt{77}}$$