Group Quiz 6
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Math 153

Name: KEY

No work = no credit

1.) Find a power series representation for $f(x) = \frac{3x^4}{25+x^2}$, its interval of convergence, and its radius of convergence.

$$f(x) = \frac{3x^4}{25+x^2} = \frac{3x^4}{25} \cdot \frac{1}{1-\left(\frac{-x^2}{25}\right)}$$

$$= \frac{3x^4}{25} \cdot \sum_{n=0}^{\infty} \left(\frac{-x^2}{25}\right)^n$$

$$= \sum_{n=0}^{\infty} \frac{3(-1)^n x^{2n+4}}{25^{n+1}}$$

I.O.C. $\left|\frac{-x^2}{25}\right| < 1$

$\Rightarrow |x^2| < 25$

$\Rightarrow |x| < 5$

since it's geometric, we know it diverges @ ± 5

R.O.C. $R = 5$

2.) Find the Taylor series generated by $f(x) = \frac{1}{x^2}$ at $a = 1$.

N	$f^{(N)}(x)$	$f^{(N)}(1)$
0	x^{-2}	1
1	$-2x^{-3}$	-2
2	$6x^{-4}$	6
3	$-24x^{-5}$	-24
$\vdots$	$\vdots$	$\vdots$
N	$(-1)^N (N+1)!\, x^{-(N+2)}$	$(-1)^N (N+1)!$

R.o.C. $R=1$

I.o.C. $0 < x < 2$

$$f(x) = \sum_{N=0}^{\infty} \frac{(-1)^N (N+1)!\,(x-1)^N}{N!}$$

$$= \sum_{N=0}^{\infty} (-1)^N (N+1)(x-1)^N$$

Ratio test:

$$\lim_{N\to\infty} \left| \frac{(N+2)(x-1)^{N+1}}{(N+1)(x-1)^N} \right| = |x-1| \lim_{N\to\infty} \frac{N+2}{N+1}$$

$$= |x-1|$$

So it converges absolutely when $|x-1| < 1$. Inspection shows it diverges otherwise

3.) You plan to estimate $\pi/4$ by evaluating the Maclaurin series for $\tan^{-1}(x)$ at $x = 1$. Use the methods developed in this course to determine how many terms of the series you would have to add to be sure the estimate is good to within 0.01 of the actual result.

$$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = \sum_{N=0}^{\infty} (-1)^N x^{2N}$$

$$\Rightarrow \tan^{-1} x = \int \frac{dx}{1+x^2}$$

$$= \sum_{N=0}^{\infty} \int (-1)^N x^{2N}\, dx$$

$$= c + \sum_{N=0}^{\infty} \frac{(-1)^N x^{2N+1}}{2N+1}$$

setting $x=0$ we find $c=0$

$$\Rightarrow \frac{\pi}{4} = \tan^{-1}(1) = \sum_{N=0}^{\infty} \frac{(-1)^N}{2N+1}$$

To find how many terms are required we use the alt. series error bound:

$$|R_k| < \left| \frac{(-1)^{k+1}}{2(k+1)+1} \right| < 0.01$$

$$\Rightarrow \frac{1}{2k+3} < 0.01$$

$$\Rightarrow 100 < 2k+3$$

$$\Rightarrow 48.5 < k$$

so k must be at least 49 (49 terms) for the required accuracy.