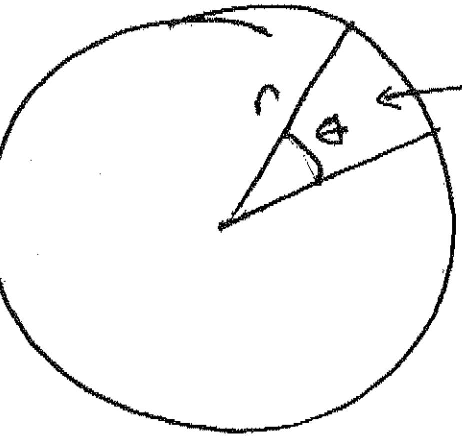
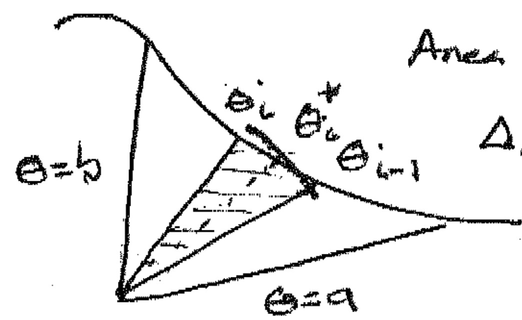
10.4: Areas and Lengths in Polar Coords



Area of the wedge

$$A = \frac{1}{2} r^2 \theta.$$



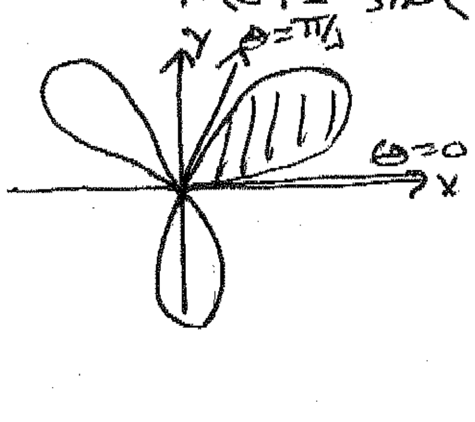
Area of wedge

$$\Delta A_i = \frac{1}{2} r(\theta_i^*) \Delta\theta$$

$$A \approx \sum_{n=1}^{N} \frac{1}{2} \left[r(\theta_i^*) \right]^2 \Delta\theta$$

AND $$A = \int_a^b \frac{1}{2} r^2 \, d\theta.$$

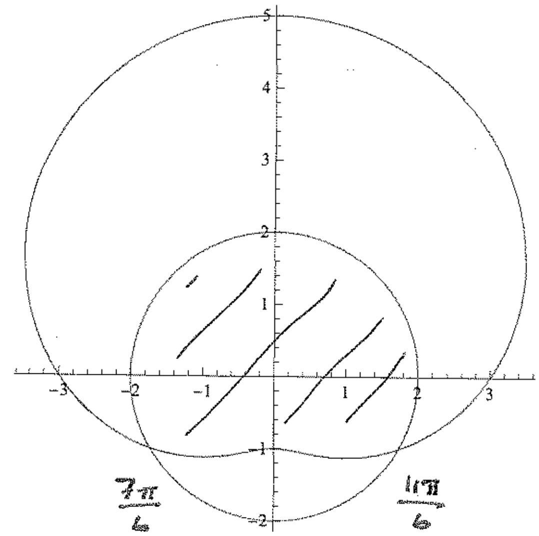
<u>Ex1</u>: Find the area of the petals of $r(\theta) = \sin(3\theta)$

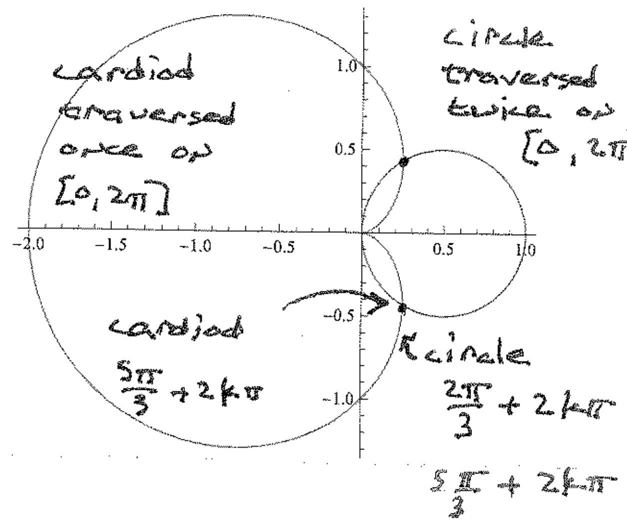


NOTE: Petal shaded on $[0, \pi/3]$

Ex 2: Find the area inside $r = 4\sin\theta$ and outside $r = 2$.

Ex 3: Find the area inside $r = 2$ and $r = 3 + 2\sin\theta$.





Ex 4: Find all pts of intersection of $r = \cos\theta$ and $r = 1 - \cos\theta$.

Recall from 10.2, that the arclength of $(x(t), y(t))$ on $a \le t \le b$ is

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

In polar, this is:

$$L = \int_{\theta=a}^{\theta=b} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta$$

$$\frac{dx}{d\theta} = r' \cos\theta - r\sin\theta$$

$$\left(\frac{dx}{d\theta}\right)^2 = \left(\frac{dr}{d\theta}\right)^2 \cos^2\theta - 2\frac{dr}{d\theta} r\sin\theta\cos\theta + r^2\sin^2\theta$$

$$\left(\frac{dy}{d\theta}\right)^2 =$$

$$\Rightarrow \left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = \left(\frac{dr}{d\theta}\right)^2 + r^2$$

so $$L = \int_a^b \sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2}\, d\theta$$

<u>Ex 5</u>: Find the exact length of $r = e^{2\theta}$ on $\theta \in [0, 2\pi]$