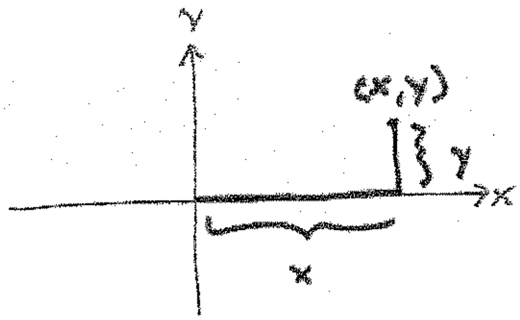
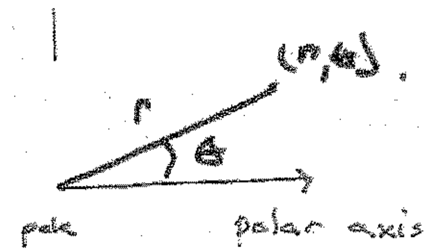
10.3 : Polar Coordinates

cartesian coords



polar coords



<u>Ex1:</u> Plot the polar point $(2, \frac{3\pi}{4})$ & find two other coords of this point. $(r>0$ & $r<0)$.

<u>Ex2:</u> Find the cartesian coords of $(r,\theta) = (-2, -5\pi/6)$.

<u>Ex3:</u> sketch the region $2 < r \leq 5$ and $\frac{3\pi}{4} < \theta < \frac{5\pi}{4}$

To convert $x = r\cos\theta$, $y = r\sin\theta$, $\tan\theta = \frac{y}{x}$
and $r^2 = x^2 + y^2$.

<u>Ex4:</u> convert $r = 2\sin\theta + 4\cos\theta$ to a cart. eqt.

$\Rightarrow r^2 = 2x + 4y$

$\Rightarrow x^2 - 2x + y^2 - 4y = 0$

$\Rightarrow (x-1)^2 + (y-2)^2 = 5$

Ex 5: sketch $r = 1 - 3\cos\theta$

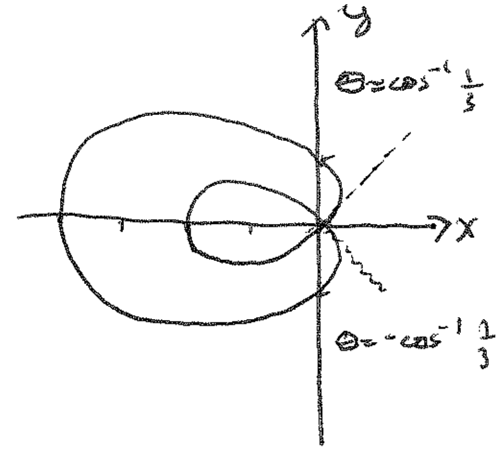
θ	r
0	-2
$\pi/2$	1
π	4
$3\pi/2$	1
2π	1

when does the curve go thru the origin?

solve $0 = 1 - 3\cos\theta$

$\Rightarrow \cos\theta = \frac{1}{3}$

$\Rightarrow \theta = \pm\cos^{-1}\left(\frac{1}{3}\right)$



see mathematica Notebook

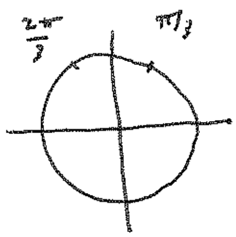
Slopes | Derivatives of Polar curves

To find $\frac{dy}{dx}$, we think of $r = f(\theta)$ as a parametric curve w/eqts: $x = f(\theta)\cos\theta$ and $y = f(\theta)\sin\theta$ with parameter θ. So w/shorthand.

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta} = \frac{r's + rc}{r'c - rs}$$

Ex 5 rev.: Find the tangent line(s) to $r = 1 - 3\cos\theta$ when $\theta = \frac{\pi}{3}$.

Slope: $$\frac{dy}{dx} = \frac{3\sin^2\theta + \cos\theta(1 - 3\cos\theta)}{3\sin\theta\cos\theta - (1 - 3\cos\theta)\sin\theta}$$



$$= \frac{\cos\theta - 3(\cos^2\theta - \sin^2\theta)}{6\sin\theta\cos\theta - \sin\theta}$$

$$= \frac{-3\cos 2\theta + \cos\theta}{-\sin\theta + 3\sin 2\theta}\Bigg|_{\theta = \frac{\pi}{3}} \quad \frac{-3\left(-\frac{1}{2}\right) + \frac{1}{2}}{-\frac{\sqrt{3}}{2} + 3\cdot\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$$

point: $x = r\cos\theta$

$$= (1-3\cos\theta)\cos\theta \Big|_{\theta=\frac{\pi}{3}} \quad (1-3(\tfrac{1}{2}))(\tfrac{1}{2}) = -\tfrac{1}{4}$$

$y = r\sin\theta$

$$= (1-3\cos\theta)\sin\theta \Big|_{\theta=\frac{\pi}{3}} \quad (-\tfrac{1}{2})(\tfrac{\sqrt{3}}{2}) = -\tfrac{\sqrt{3}}{4}$$

tangent line: $y + \frac{\sqrt{3}}{4} = \frac{2}{\sqrt{3}}(x + \frac{1}{4})$

Ex 5 rev. again (and easier): Find where $r = 1 - 3\cos\theta$ has horizontal and vertical tangents. (find θ)

vertical tangents: solve $\frac{dy}{d\theta} = 0$

$$\Rightarrow 0 = 6\sin\theta\cos\theta - \sin\theta$$

$$\Rightarrow 0 = \sin\theta(6\cos\theta - 1)$$

$$\Rightarrow \theta = 0,\ \pi,\ \pm\cos^{-1}(\tfrac{1}{6})$$

horizontal tangents: solve $\frac{dx}{d\theta} = 0$

$$\Rightarrow 0 = \cos\theta - 3\cos 2\theta$$

$$\Rightarrow 0 = \cos\theta - 3(2\cos^2\theta - 1)$$

$$\Rightarrow \quad = -6\cos^2\theta + \cos\theta + 3$$

$$\Rightarrow 0 = 6\cos^2\theta - \cos\theta - 3$$

$$\Rightarrow \cos\theta = \frac{1 \pm \sqrt{1 - 4(6)(3)}}{12}$$

$$\Rightarrow \cos\theta = \frac{1 \pm \sqrt{73}}{12}$$

$$\Rightarrow \theta = \pm\cos^{-1}\left(\frac{1 \pm \sqrt{73}}{12}\right)$$