Test 3a
Dusty Wilson
Math 151

No work = no credit

No Symbolic Calculators

Name: Key

To divide a cube into two other cubes, a fourth power or in general any power whatever into two powers of the same denomination above the second is impossible, and I have assuredly found an admirable proof of this, but the margin is too narrow to contain it.

Pierre de Fermat (1601 - 1665)
French mathematician

Warm-ups (1 pt each): $\frac{d}{dx}\frac{1}{x} = -\frac{1}{x^2}$ $\qquad \frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$ $\qquad \frac{d}{dx}\frac{1}{\sqrt{x}} = -\frac{1}{2x^{3/2}}$

1.) (1 pt) According to Fermat (see above), how long do you think his admirable proof was?

He thought it was going to be easy.

2.) (4 pts) Evaluate $\lim_{x\to\infty} 3x\sin\left(\frac{1}{x}\right) \to \infty \cdot 0$

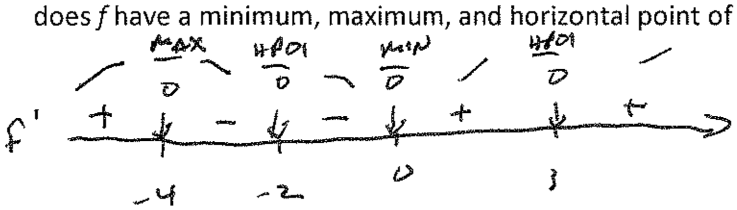
$$= \lim_{x\to\infty} \frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{3x}} \to \frac{0}{0}$$

$$\overset{H}{=} \lim_{x\to\infty} \frac{\cos\left(\frac{1}{x}\right)\cdot\frac{-1}{x^2}}{\frac{-1}{3x^2}}$$

$$= \lim_{x\to\infty} 3\cos\left(\frac{1}{x}\right)$$

$$= 3$$

3.) (4 pts) Suppose the derivative of $f(x)$ is $f'(x) = 3x^3(x+4)(x+2)^2(x-3)^4$. At what *x* value(s) does *f* have a minimum, maximum, and horizontal point of inflection?



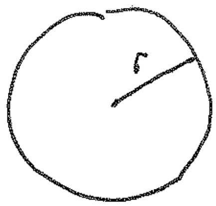
Minimum: $x = 0$

Maximum: $x = -4$

Horizontal Point of Inflection: $x = -2$ and $x = 3$

4.) (4 pts) A rock is thrown into a still pond. The circular ripples move outward from the point of impact of the rock so that the radius of the circle formed by a ripple increases at the rate of 3 feet per second. Find the rate at which the area is changing at the instant the radius is 6 feet.

① Draw a pic



② relate variables

$A = \pi r^2$

③ we know: $\frac{dr}{dt} = 3$

④ we want: $\frac{dA}{dt}$

⑤ implicit

$\frac{d}{dt} A = \frac{d}{dt} \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ $\Big|_{\substack{\frac{dr}{dt} = 3 \\ r = 6}}$

$2\pi(6)(3) = 36\pi$

⑥ Answer

The area is changing @ 36π ft^2/s.

5.) (4 pts) Evaluate $\lim_{x \to \infty} \left(1+\frac{3}{x}\right)^{2x}$

$$= \lim_{x\to\infty} e^{\ln\left(1+\frac{3}{x}\right)^{2x}}$$

$$= \lim_{x\to\infty} e^{2x \ln\left(1+\frac{3}{x}\right)}$$

$$= \lim_{x\to\infty} e^{\frac{\ln\left(1+\frac{3}{x}\right)}{\frac{1}{2x}}} \quad \begin{matrix}\to 0 \\ \overline{\to 0}\end{matrix}$$

$$\overset{H}{=} \lim_{x\to\infty} e^{\frac{\frac{1}{1+\frac{3}{x}} \cdot \frac{-3}{\cancel{x^2}}}{\frac{1}{2} \cdot \frac{-1}{\cancel{x^2}}}}$$

$$= \lim_{x\to\infty} e^{\frac{+6}{1+3/x}}$$

$$= e^{+6}$$

Test 3
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French mathematician

1.) (4 pts) Find and clearly label all local and absolute extremes of $f(x) = x^2 + 2x - 15$ on $[-4, 4]$.

$f'(x) = 2x + 2$

$= 2(x+1)$

f': $-$ min 0 at $x=-1$, $+$

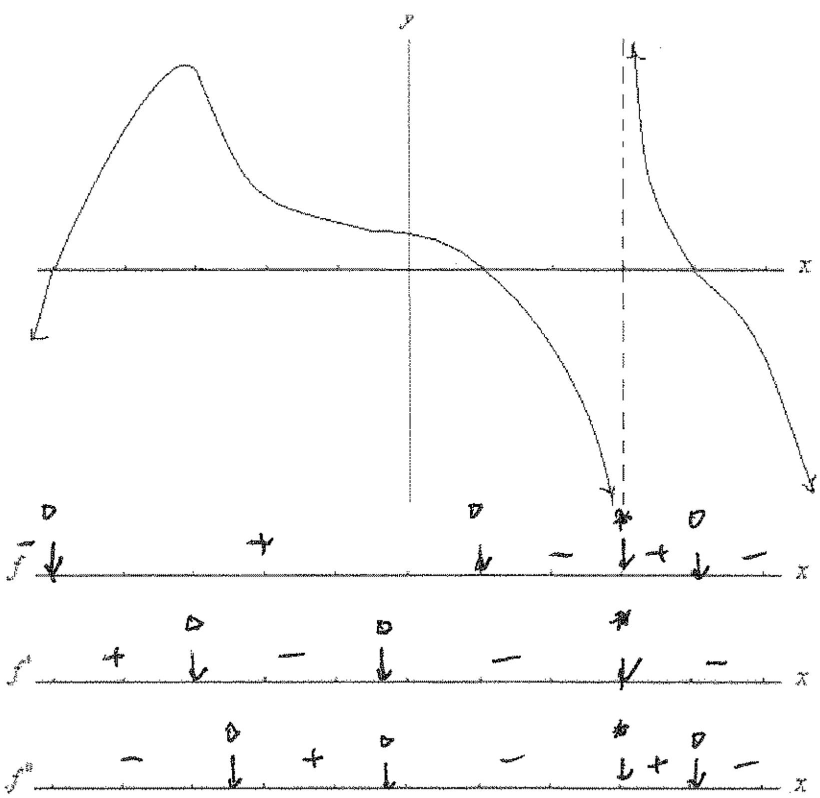
check

$x = -4 \to (-4, -7)$

$x = -1 \to (-1, -16)$ Abs min, local min

$x = 4 \to (4, 9)$ Abs max

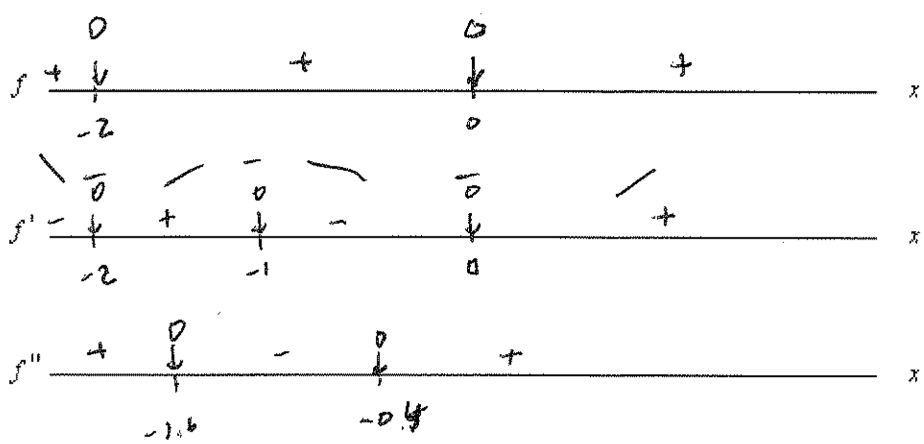
2.) (6 pts) Use the graph if f to complete the sign diagrams of f and its first and second derivative.

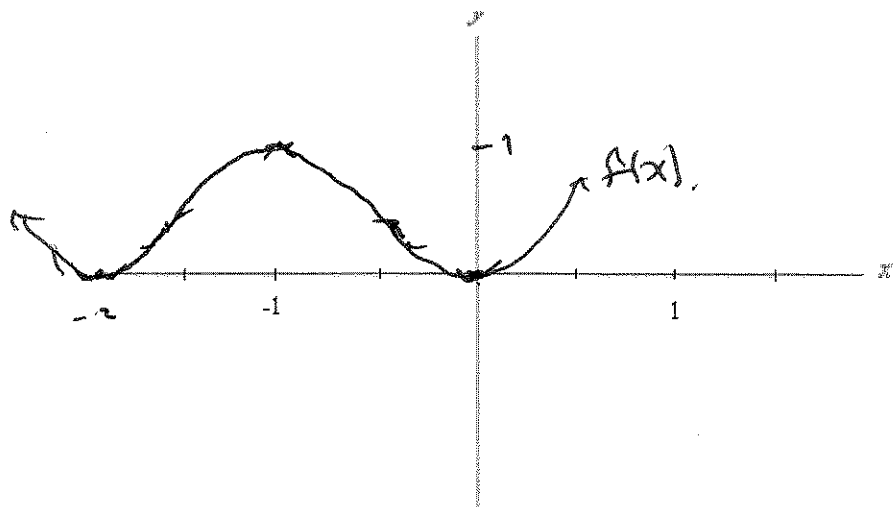


3.)(6 pts) Use <u>calculus</u> to clearly and carefully sketch a graph of $f(x)$.

$$f(x)=x^4+4x^3+4x^2=x^2(x+2)^2 \qquad f(-1)=1+-4+4=1$$
$$f'(x)=4x^3+12x^2+8x=4x(x+1)(x+2)$$
$$f''(x)=12x^2+24x+8\approx 12(x+0.4)(x+1.6)$$

Find and label all x-intercepts, extrema (including y-values), and points of inflection. (only x-vals)





4.) (4 pts) Approx $\frac{1}{103}$. write as a decimal. Find the tangent line to $y=\frac{1}{x}$ @ $x=100$ and use to approx when $x=103$.

point: $\left(100, \frac{1}{100}\right)$

slope: $-\frac{1}{x^2}\Big|_{x=100} = -\frac{1}{10000}$

tangent: $y-\frac{1}{100} = -\frac{1}{10000}(x-100)\Big|_{x=103}$

$\Rightarrow y = 0.01 - 0.0003$

$= 0.0097$