high 100%
$\bar{X} = 80.6\%$
med = 82.6%

100	90's	80s	70's	60's	< 60
1	卌 III	卌 II	卌 II	III	II

Test 2A
Dusty Wilson
Math 151

Name: Key

We are usually convinced more easily by reasons we have found ourselves than by those which have occurred to others.

Blaise Pascal (1623 - 1662)
French mathematician

No work = no credit

No Sineful Calculators

Warm-ups (1 pt each): $\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$ $\quad \frac{d}{dx}\arccos(x) = \frac{-1}{\sqrt{1-x^2}}$ $\quad \frac{d}{dx}\tan(x) = \sec^2(x)$

1.) (1 pt) According to Pascal (see above), what kind of logic do we find most compelling?

We are most motivated by our own discoveries.

2.) (6 pts) Differentiate $h(x) = 8x^7 + 6\sqrt[5]{x} + \frac{4}{x^3} + 2\log_9(x) + 8^x + 7\tan^{-1}(x)$. Express your answer as an equation.

$$\Rightarrow h'(x) = 56x^6 + \frac{6}{5}x^{-4/5} + \frac{2}{x\ln 9} + 8^x \ln 8 + \frac{7}{1+x^2}$$

$-12x^{-4}$

3.) (4 pts) Find $\frac{d}{dx}3x^4\sec(x)$. Simplification is optional.

$$= 12x^3\sec(x) + 3x^4 \sec x \tan x$$

4.) (4 pts) If $g(x) = \frac{4\sqrt[3]{x}}{5x^2 - 6x + 7}$ find $g'(x)$. Simplification is optional.

$$\Rightarrow g'(x) = \frac{\frac{4}{3}x^{-2/3}(5x^2 - 6x + 7) - (10x - 6)(4\sqrt[3]{x})}{(5x^2 - 6x + 7)^2}$$

5.) (4 pts) If $f(x) = 2\tan^{-1}\left(e^{x^5+3}\right)$ find $\frac{df}{dx}$. Simplification is optional.

$$\Rightarrow \frac{df}{dx} = \frac{2}{1+\left(e^{x^5+3}\right)^2} \cdot e^{x^5+3} \cdot 5x^4$$

6.) (4 pts) Find the equation of the second derivative of $\overbrace{y = e^{3x^5-2x}}^{\text{position}}$. Simplification is optional.

$$\Rightarrow y' = e^{3x^5-2x} \cdot (15x^4 - 2)$$

$$\Rightarrow y'' = e^{3x^5-2x}(15x^4-2)^2 + e^{3x^5-2x}(60x^3)\Big|_{x=0} \quad 4$$

acceleration: 4 m/s

7.) (4 pts) Differentiate $y = (\cos(x))^{x^3}$. Express y' in terms of x. Simplification is optional.

$$\Rightarrow \ln y = x^3 \ln(\cos x)$$

$$\Rightarrow \frac{d}{dx} \ln y = \frac{d}{dx} x^3 \ln(\cos x)$$

$$\Rightarrow \frac{y'}{y} = 3x^2 \ln(\cos x) + x^3 \frac{-\sin x}{\cos x}$$

$$\Rightarrow y' = x^3 \ln(\cos x) \left[3x^2 \ln(\cos x) - x^3 \tan x\right]$$

Test 2A
Dusty Wilson
Math 151

Name: ______________________ **Number:** _____

It is rare to find learned men who are clean, do not stink and have a sense of humour.

Gottleb Leibnez (1646 - 1716)
German mathematician

No work = no credit

No Symbolic Calculators

1.) (4 pts) If $q=\dfrac{\cos(7z)\sqrt{3-4z}}{\left(5z^6-1\right)^7}$, find $\dfrac{dq}{dz}$. Give your answer as an equation. Simplification is optional.

$$\Rightarrow \ln q = \ln\left(\frac{\cos 7z\sqrt{3-4z}}{(5z^6-1)^7}\right)$$

$$\Rightarrow \frac{d}{dz}\ln q = \frac{d}{dz}\left(\ln(\cos 7z) + \frac{1}{2}\ln(3-4z) - 7\ln(5z^6-1)\right)$$

$$\Rightarrow \frac{q'}{q} = \frac{-7\sin 7z}{\cos 7z} + \frac{1}{2}\,\frac{-4}{3-4z} - 7\,\frac{30z^5}{5z^6-1}$$

$$\Rightarrow q' = \frac{\cos(7z)\sqrt{3-4z}}{(5z^6-1)^7}\left(-7\tan(7z) - \frac{2}{3-4z} - \frac{210z^5}{5z^6-1}\right)$$

Alt soln:

$$q' = \frac{\left(-7\sin(7z)\sqrt{3-4z} + \frac{1}{2}(3-4z)^{-1/2}(-4)\cos 7z\right)(5z^6-1)^7 - 7(5z^6-1)^6\cdot 30z^5\cos(7z)\sqrt{3-4z}}{\left[(5z^6-1)^7\right]^2}$$

2.) (4 pts) Find the equation of the normal line to $x^2+y^2=5-2xy+4x$ at the point $(1,2)$

$$\Rightarrow \frac{d}{dx}(x^2+y^2) = \frac{d}{dx}(5-2xy+4x)$$

$$\Rightarrow 2x + 2y\frac{dy}{dx} = -2y - 2x\frac{dy}{dx} + 4$$

$$\Rightarrow \frac{dy}{dx}(2x+2y) = -2x - 2y + 4$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x+y-2}{x+y}\Bigg|_{(x,y)=(1,2)} = -\frac{1+2-2}{1+2} = -\frac{1}{3}$$

perpendicular lines

Normal line: $y-2=3(x-1)$

3.) (4 pts) Find and evaluate the following given the table below. Circle your answers.

a.) $\dfrac{d}{dx}\left[\dfrac{g(x)}{f(x)}\right]$ when $x=2$

x	1	2	3	4	5
$f(x)$	5	3	1	2	4
$f'(x)$	5	2	4	3	1
$g(x)$	4	1	2	3	5
$g'(x)$	1	5	4	2	3

$= \dfrac{g'(2)\,f(2) - f'(2)\,g(2)}{(f(2))^2}$

$= \dfrac{5(3) - 2(1)}{3^2}$

$= \dfrac{13}{9}$

b.) $\dfrac{d}{dx} g\left[f\left(x^2\right)\right]$ when $x=2$

$= g'(f(x^2))\,f'(x^2)\cdot 2x$

$= g'(f(4))\,f'(4)\,2(2)$

$= g'(2)(3)(4)$

$= 5(12)$

$= 60$

4.) (4 pts) Differentiate $y = 3\left(\dfrac{x^2}{\sec(4x)}\right)^{-5}$ with respect to x. Simplification is optional.

$y' = -15\left(\dfrac{x^2}{\sec 4x}\right)^{-6}\left(\dfrac{2x\sec(4x) - \sec(4x)\tan(4x)\cdot 4\cdot x^2}{\sec^2(4x)}\right)$

Test 2B
Dusty Wilson
Math 151

Name: Kay

We are usually convinced more easily by reasons we have found ourselves than by those which have occurred to others.

Blaise Pascal (1623 - 1662)
French mathematician

No work = no credit

No Sineful Calculators

Warm-ups (1 pt each): $\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$ $\quad \frac{d}{dx}\arccos(x) = \frac{-1}{\sqrt{1-x^2}}$ $\quad \frac{d}{dx}\tan(x) = \sec^2 x$

1.) (1 pt) According to Pascal (see above), what kind of logic do we find most compelling?

Self-discovery beats lecture hands down.

2.) (6 pts) Differentiate $h(x) = 7x^6 + 5\sqrt[4]{x} + \frac{3}{x^2} + 9\log_8(x) + 7^x + 6\tan^{-1}(x)$. Express your answer as an <u>equation</u>.

$$h'(x) = 42x^5 + \frac{5}{4}x^{-\frac{3}{4}} - \frac{6}{x^3} + \frac{9}{x\ln 8} + 7^x \ln 7 + \frac{6}{1+x^2}$$

3.) (4 pts) Find $\frac{d}{dx}5x^6\csc(x)$. Simplification is optional.

$$= 30x^5 \csc x - 5x^6 \csc x \cot x$$

4.) (4 pts) If $g(x) = \frac{5\sqrt{x}}{6x^3 - 7x + 8}$ find $g'(x)$. Simplification is optional.

$$\Rightarrow g'(x) = \frac{\frac{5}{2}x^{-\frac{1}{2}}(6x^3 - 7x + 8) - 5\sqrt{x}\,(18x^2 - 7)}{(6x^3 - 7x + 8)^2}$$

$\overline{18}$ $\qquad$ $\overline{46}$

5.) (4 pts) If $f(x) = 3\tan^{-1}\left(e^{x^6+4}\right)$ find $\frac{df}{dx}$. Simplification is optional.

$$\frac{df}{dx} = \frac{3}{1+(e^{x^6+4})^2} \cdot e^{x^6} \cdot 6x^5$$

$$= \frac{18x^5 e^{x^6}}{1+(e^{x^6+4})^2}$$

6.) (4 pts) The position in meters of a particle after t seconds is given by $s = e^{4t^6-7t}$. Find the acceleration of the particle at $t = 0$.

$$\Rightarrow s' = V = e^{4t^6-7t} \cdot (24t^5 - 7)$$

$$\Rightarrow v' = a = e^{4t^6-7t}(24t^5-7)^2 + e^{4t^6-7t}(120t^4)\Big|_{t=0} \quad 49$$

The accalaration is $49\ m/s^2$

7.) (4 pts) Differentiate $y = (\sin(x))^{x^4}$. Express y' in terms of x. Simplification is optional.

$$\Rightarrow \ln y = x^4 \ln(\sin x)$$

$$\Rightarrow \frac{d}{dx}\ln y = \frac{d}{dx} x^4 \ln(\sin x)$$

$$\Rightarrow \frac{y'}{y} = 4x^3\ln(\sin x) + x^4 \frac{\cos x}{\sin x}$$

$$\Rightarrow y' = (\sin x)^{x^4}\left(4x^3\ln(\sin x) + x^4\cot x\right)$$

12

Test 2B
Dusty Wilson
Math 151

Name: ______________________ **Number:** _____

It is rare to find learned men who are clean, do not stink and have a sense of humour.

Gottleb Leibnez (1646 - 1716)
German mathematician

No work = no credit

No Symbolic Calculators

1.) (4 pts) Differentiate $y=4\left(\dfrac{x^3}{\tan(5x)}\right)^{-6}$ with respect to x. Simplification is optional.

$$\Rightarrow y' = -24\left(\frac{x^3}{\tan 5x}\right)^{-7} \cdot \frac{3x^2\tan 5x - 5\sec^2 5x \cdot x^3}{\tan^2 5x}$$

Alt soln.

$$\ln y = \ln\left(4\left(\frac{x^3}{\tan 5x}\right)^{-6}\right) = \ln 4 - 18\ln x + 6\ln\tan 5x$$

$$\Rightarrow \frac{y'}{y} = -\frac{18}{x} + \frac{30\sec^2 5x}{\tan 5x}$$

$$\Rightarrow y' = 4\left(\frac{x^3}{\tan 5x}\right)^{-6}\left(\frac{30\sec^2 5x}{\tan 5x} - \frac{18}{x}\right)$$

2.) (4 pts) If $q=\dfrac{\sin(6z)\sqrt{4-5z}}{(6z^7-1)^8}$, find $\dfrac{dq}{dz}$. Give your answer as an equation. Simplification is optional.

$$\Rightarrow \frac{d}{dz}\ln q = \frac{d}{dz}\left(\ln(\sin 6z) + \frac{1}{2}\ln(4-5z) - 8\ln(6z^7-1)\right)$$

$$\Rightarrow \frac{q'}{q} = \frac{6\cos 6z}{\sin 6z} + \frac{1}{2}\frac{-5}{4-5z} - 8\frac{42z^6}{6z^7-1}$$

$$\Rightarrow \frac{dq}{dz} = \frac{\sin 6z\sqrt{4-5z}}{(6z^7-1)^8}\left(6\cot 6z - \frac{5/2}{4-5z} - \frac{336z^6}{6z^7-1}\right)$$

Alt. soln.

$$q' = \frac{\left(6\cos(6z)\sqrt{4-5z} + \sin(6z)\cdot\frac{1}{2}(4-5z)^{-\frac{1}{2}}\cdot(-5)\right)(6z^7-1)^8 - 8(6z^7-1)^7\cdot 42z^6\cdot\sin(6z)\sqrt{4-5z}}{\left[(6z^7-1)^8\right]^{16}}$$

8

3.) (4 pts) Find the equation of the normal line to $x^2 + y^2 = 9 + 2xy - 4x$ at the point $(2,1)$.

$$\Rightarrow 2x + 2y \frac{dy}{dx} = 2y + 2x \frac{dy}{dx} - 4$$

$$\Rightarrow \frac{dy}{dx}(2y - 2x) = 2y - 2x - 4$$

$$\Rightarrow \frac{dy}{dx} = \frac{2y - 2x - 4}{2y - 2x} \Bigg|_{(x,y) = (2,1)} \quad \frac{-6}{-2} = 3$$

normal line

$$y - 1 = -\frac{1}{3}(x - 2)$$

4.) (4 pts) Find and evaluate the following given the table below. Circle your answers.

x	1	2	3	4	5
$f(x)$	5	3	1	2	4
$f'(x)$	5	2	4	3	1
$g(x)$	4	1	2	3	5
$g'(x)$	1	5	4	2	3

a.) $\frac{d}{dx}\left[\frac{g(x)}{f(x)}\right]$ when $x = 2$

$$= \frac{g'(2)f(2) - f'(2)g(2)}{[f(2)]^2}$$

$$= \frac{5(3) - 2(1)}{3^2}$$

$$= 13/9$$

b.) $\frac{d}{dx} g\left[f(x^2)\right]$ when $x = 2$

$$= g'(f(x^2)) \cdot f'(x^2) \cdot 2x$$

$$= g'(\underbrace{f(4)}_{2})\, \underbrace{f'(4)}_{3} \cdot 4$$

$$= \underbrace{g'(2)}_{5}(3)(4)$$

$$= 60$$

8