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Test 1 A
Dusty Wilson
Math 151

Name: key Number: III

It is rare to find learned men who are clean, do not stink and have a sense of humour.

No work = no credit

Gottlieb Leibnez (1646 - 1716)
German mathematician

No Sineful Calculators

Warm-ups (1 pt each): $-2^2 = \underline{-4}$ $-1^0 = \underline{-1}$ $2+2 = \underline{4}$

1.) (1 pt) The quote (above) was said about Leibniz. What were two of his positive qualities?

He was clean & funny.

2.) (4 pts) Using the methods developed in the course, evaluate $\lim_{x \rightarrow \infty} \frac{3x^2 - 2x + 1}{4 + 5x - 7x^2}$

$$\lim_{x \rightarrow \infty} \frac{3x^2 - 2x + 1}{4 + 5x - 7x^2} = \lim_{x \rightarrow \infty} \frac{3 - \frac{2}{x} + \frac{1}{x^2}}{\frac{4}{x^2} + \frac{5}{x} - 7} = -\frac{3}{7}$$

3.) (4 pts) Consider the function $f(x) = 4x^2 - 3x + 5$. Use the definition of the derivative to find the derivative of f . Hint: You may check using the techniques of chapter 3.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{4(x+h)^2 - 3(x+h) + 5 - (4x^2 - 3x + 5)}{h} \\ &= \lim_{h \rightarrow 0} \frac{4x^2 + 8xh + 4h^2 - 3x - 3h + 5 - 4x^2 + 3x - 5}{h} \\ &= \lim_{h \rightarrow 0} (8x + 4h - 3) \\ &= 8x - 3 \end{aligned}$$

4.) (5 pts) Use the precise definition of the limit to prove $\lim_{x \rightarrow 4} (3x+2) = 14$

claim: $\lim_{x \rightarrow 4} (3x+2) = 14$

proof,

Let $\epsilon > 0$ be given

choose $\delta = \frac{\epsilon}{3}$

If $0 < |x-4| < \delta$

$\Rightarrow |x-4| < \frac{\epsilon}{3}$

$\Rightarrow 3|x-4| < \epsilon$

$\Rightarrow |3x-12| < \epsilon$

$\Rightarrow |(3x+2)-14| < \epsilon$

Hence $\lim_{x \rightarrow 4} (3x+2) = 14$.

5.) (4 pts) Evaluate $\lim_{x \rightarrow 9} \frac{x-9}{\sqrt{x}-3} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3}$

$$= \lim_{x \rightarrow 9} \frac{(x/9)(\sqrt{x}+3)}{x/9}$$

$$= 6$$

6.) (4 pts) Find the equation of the tangent line to $f(x) = x^2$ when $x = -3$. using any method.

point: $(-3, 9)$

slope: $f'(x) = 2x \Big|_{x=-3} = -6$

tangent line: $y-9 = -6(x+3)$

$\Rightarrow y = -6x - 9$

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No Symbolic Calculators

1.) (4 pts) Evaluate $\lim_{x \rightarrow \infty} \frac{\sqrt{16x^8 + 2x} - 5}{2 - 3x^4}$ = $\lim_{x \rightarrow \infty} \frac{\sqrt{16 + \frac{2}{x^7}} - \frac{5}{x^4}}{\frac{2}{x^4} - 3} = \frac{4}{-3} = -\frac{4}{3}$

2.) (5 pts) The table gives the values of f near 2, but not equal to 2. Use the table to estimate $\lim_{x \rightarrow 2} f(x)$.

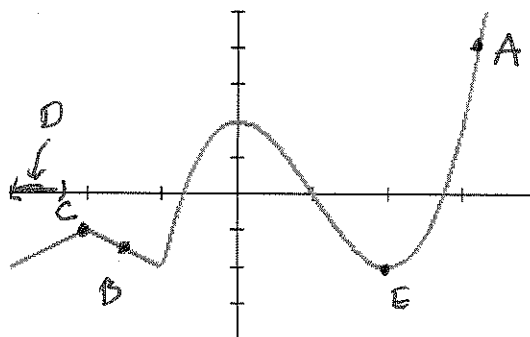
x	1	1.9	1.99	2	2.01	2.1	3
$f(x)$	0.7829	1.0858	1.1051	1.107	1.1092	1.1269	1.2565

↑ midpoint.

3.) (5 pts) Consider the graph of f (right).

- Find and label a point A where f' is positive.
- Find and label a point B where f' is negative.
- Find and label a point C where f' is undefined.
- Find and label an interval D where f' is constant.
- Find and label a point E where f' is zero and is negative immediately to the left of E and positive to the right of E

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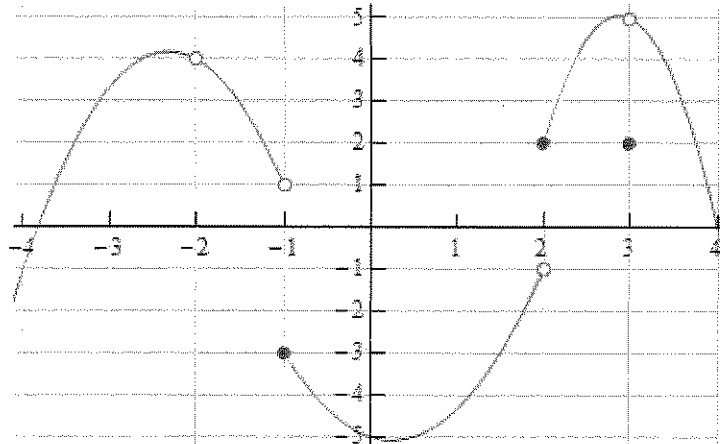
4.) (4 pts) Consider the graph of g .

- a.) For what value(s) a does $\lim_{x \rightarrow a} g(x)$ exist but g fail to be continuous?

$$a = -2, 3$$

- b.) At what value(s) a is g continuous from the right?

$$a = -1, 2$$



5.) (3 pts) Suppose the height of a falling object after t seconds is given by the function $s(t) = 180 - 16t^2$ where the position is given in feet above the ground.

- a.) Find and interpret $s(3) = 36$

The object is 36 ft off the ground after 3 seconds.

- b.) Use any method to find $s'(3)$

$$s'(t) = -32t \Big|_{t=3} = -96$$

- c.) Interpret $s'(3)$ including units

After 3 sec, the object is falling @ 96 ft/s

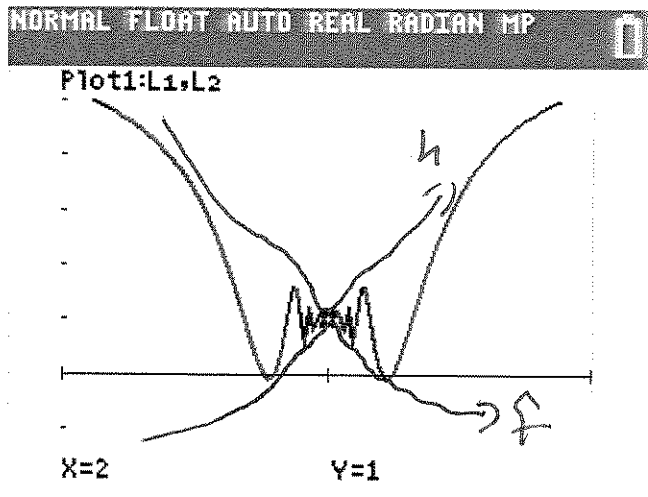
6.) (4 pts) Explain how you might use the Squeeze Theorem to prove that for the function g (pictured below) the $\lim_{x \rightarrow 2} g(x) = 1$

Find functions f and h
 s.t. $f(x) \leq g(x) \leq h(x)$
 near $x=2$ w/

$$\lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} h(x) = 1$$

This would prove

$\lim_{x \rightarrow 2} g(x) = 1$ using the
 Squeeze thm.



7.) (5 pts) Use the graph of g to answer the questions below.

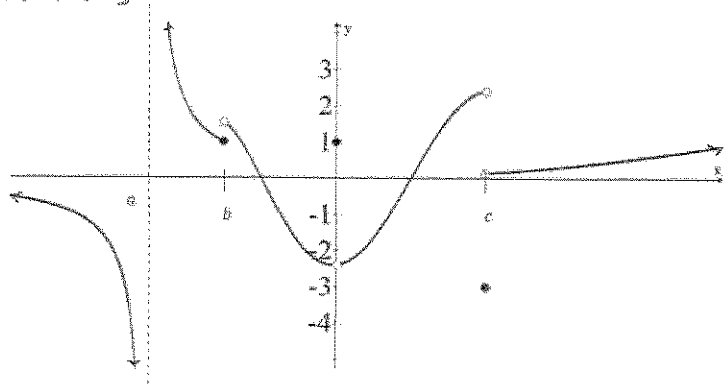
a.) $\lim_{x \rightarrow 0} g(x) = -2.5$ (estimate)

b.) Find $\lim_{x \rightarrow b^-} g(x) = 1$

c.) Find $\lim_{x \rightarrow b} g(x) = DNE$

d.) Find $\lim_{x \rightarrow \infty} g(x) = 0$

e.) Find $\lim_{x \rightarrow a^+} g(x) = \infty$



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No Sineful Calculators

Warm-ups (1 pt each): $-2^2 = \underline{-4}$ $-1^0 = \underline{-1}$ $2+2 = \underline{4}$

1.) (1 pt) The quote (above) was said about Leibniz. What were two of his positive qualities?

He was clean & funny.

2.) (4 pts) Using the methods developed in the course, evaluate $\lim_{x \rightarrow \infty} \frac{2x^3 - 2x + 1}{4 + 5x - 11x^3}$

$$\lim_{x \rightarrow \infty} \frac{2x^3 - 2x + 1}{4 + 5x - 11x^3} = \lim_{x \rightarrow \infty} \frac{2 - \frac{2}{x^2} + \frac{1}{x^3}}{\frac{4}{x^3} + \frac{5}{x^2} - 11} = -\frac{2}{11}$$

5.) (4 pts) Consider the function $f(x) = 5x^2 - 7x + 5$. Use the definition of the derivative to find the derivative of f . Hint: You may check using the techniques of chapter 3.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{5(x+h)^2 - 7(x+h) + 5 - (5x^2 - 7x + 5)}{h} \\ &= \lim_{h \rightarrow 0} \frac{5x^2 + 10xh + 5h^2 - 7x - 7h + 5 - 5x^2 + 7x - 5}{h} \\ &= \lim_{h \rightarrow 0} (10x + 5h - 7) \\ &= 10x - 7 \end{aligned}$$

3.) (5 pts) Use the precise definition of the limit to prove $\lim_{x \rightarrow 5} (3x + 4) = 19$

claim: $\lim_{x \rightarrow 5} (3x + 4) = 19$

proof

Let $\epsilon > 0$ be given.

Choose $\delta = \frac{\epsilon}{3}$

If $0 < |x - 5| < \delta$

$\Rightarrow |x - 5| < \frac{\epsilon}{3}$

$\Rightarrow 3|x - 5| < \epsilon$

$\Rightarrow |3x - 15| < \epsilon$

$\Rightarrow |(3x + 4) - 19| < \epsilon$

Hence $\lim_{x \rightarrow 5} (3x + 4) = 19$

4.) (4 pts) Evaluate $\lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$

$= \lim_{x \rightarrow 4} \frac{(x-4)(\sqrt{x}+2)}{x+4}$

$= 4$

5.) (4 pts) Find the equation of the tangent line to $f(x) = x^2$ when $x = -4$ using any calculus method.

point
 $(-4, 16)$

slope
 $f'(x) = 2x \Big|_{x=-4} = -8$

tangent line

$y - 16 = -8(x + 4)$

$\Rightarrow y = -8x - 16$

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 stink and have a sense of humour.*

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No Symbolic Calculators

1.) (4 pts) Evaluate $\lim_{x \rightarrow \infty} \frac{\sqrt{25x^{10} + 2x} - 7}{2 - 4x^5}$

$$= \lim_{x \rightarrow \infty} \frac{\sqrt{25 + \frac{2}{x^9}} - \frac{7}{x^5}}{\frac{2}{x^5} - 4} = -\frac{5}{2}$$

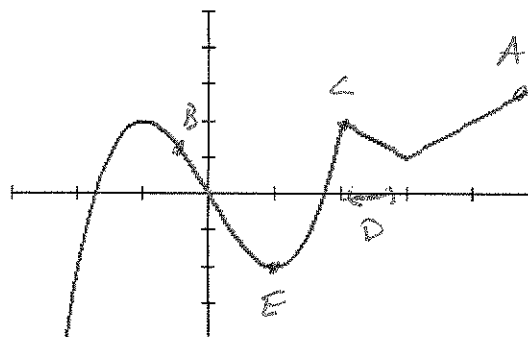
2.) (2 pts) The table gives the values of f near 2, but not equal to 2. Use the table to estimate $\lim_{x \rightarrow 2} f(x)$.

x	1	1.9	1.99	2	2.01	2.1	3
$f(x)$	2.0174	2.3203	2.3396	2.342	2.3437	2.3614	2.491

3.) (5 pts) Consider the graph of f (right).

- Find and label a point A where f' is positive.
- Find and label a point B where f' is negative.
- Find and label a point C where f' is undefined.
- Find and label an interval D where f' is constant.
- Find and label a point E where f' is zero and is negative immediately to the left of E and positive to the right of E.

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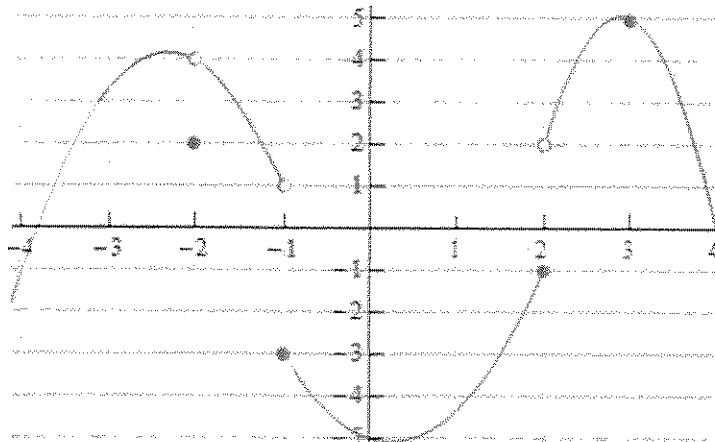
4.) (4 pts) Consider the graph of g .

- a.) For what value(s) a does $\lim_{x \rightarrow a} g(x)$ exist but g fail to be continuous?

$$a = -2$$

- b.) At what value(s) a is g continuous from the right?

$$a = -1$$



5.) (3 pts) Suppose the height of a falling object after t seconds is given by the function $s(t) = 80 - 16t^2$ where the position is given in feet above the ground.

- a.) Find and interpret $s(2) = 16$

The object is 16 ft up after 2 seconds

- b.) Use any method to find $s'(2)$

$$s'(t) = -32t \Big|_{t=2} = -64$$

- c.) Interpret $s'(2)$ including units

The object is falling @ 64 ft/s after 2 seconds.

6.) (4 pts) Explain how you might use the Squeeze Theorem to prove that for the function g (pictured below) the $\lim_{x \rightarrow 2} g(x) = -1$

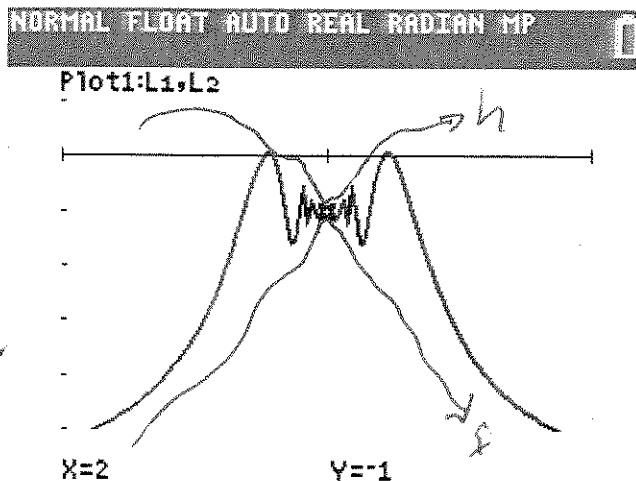
Find f & h sit. 1 pt.

$$f(x) \leq g(x) \leq h(x) \text{ near } x=2 \text{ and}$$

$$\lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} h(x) = -1.$$

This would prove

$\lim_{x \rightarrow 2} g(x) = -1$ by the Squeeze theorem.



7.) (5 pts) Use the graph of g to answer the questions below.

a.) $\lim_{x \rightarrow 1} g(x) = \underline{1}$

b.) Find $\lim_{x \rightarrow -1^+} g(x) = \underline{0.75}$ (estimate)

c.) Find $\lim_{x \rightarrow 0} g(x) = \underline{DNE}$

d.) Find $\lim_{x \rightarrow -\infty} g(x) = \underline{-1}$

e.) Find $\lim_{x \rightarrow 0^-} g(x) = \underline{\infty}$

