Test 1 – Version E
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Math 151

Name: Key.

God may not play dice with the universe, but something strange is going on with the prime numbers.

Paul Erdös (1913 - 1996)
Hungarian mathematician

No work = no credit

No Symbolic Calculators

Warm-ups (1 pt each): $-4^2 =$ -16 $\qquad -4^0 =$ -1 $\qquad \sqrt{(-4)^2} =$ 4

1.) (1 pt) According to Erdös (see above), where is something unusual taking place?

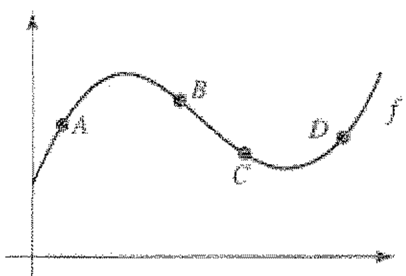
Prime numbers are mysterious.

2.) If $f(1) = 2$ and the average rate of change of f from $x = 1$ to $x = 5$ is 3, what is $f(5)$?

$$3 = \frac{f(5) - f(1)}{5 - 1} \Rightarrow 12 = f(5) - 2 \Rightarrow f(5) = 14$$

2/5 mercy pts.
3/5 for $f(5) = 5$

3.) Consider the graph of f.



a.) Between which consecutive labeled points is f' negative?

between B & C

b.) Between which consecutive labeled points does the sign of f' change from negative to positive?

between C & D

4.) (10 pts) Evaluate $\lim\limits_{x\to-2} \dfrac{x+2}{\sqrt{x^2+5}-3} \cdot \dfrac{\sqrt{x^2+5}+3}{\sqrt{x^2+5}+3}$ ← 5 pts.

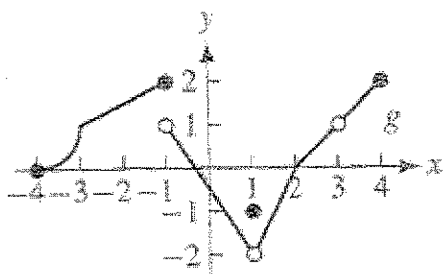
$= \lim\limits_{x\to-2} \dfrac{(x+2)(\sqrt{x^2+5}+3)}{x^2+5-9}$ ← 6 pts.

$= \lim\limits_{x\to-2} \dfrac{\cancel{(x+2)}(\sqrt{x^2+5}+3)}{\cancel{(x+2)}(x-2)}$

$= \dfrac{6}{-4}$

$= -\dfrac{3}{2}$

5.) (7 pts) Use the graph of g to answer the questions below.



a.) (1 pt) Find $g(4)$ $= 2$

b.) Find $\lim\limits_{x\to3} g(x)$ $= 1$

c.) Find $\lim\limits_{x\to-1^+} g(x)$ $= 1$

d.) Find $g'(0)$ $= -\frac{3}{2}$

e.) (2 pts) Is g continuous at $x=1$? Explain why or why not using the definition.

No, since $\lim\limits_{x\to1} g(x) \neq g(1)$

6.) (10 pts) Use the definition to find the derivative of $f(x) = 5x^2 + 3x - 7$

$$f'(x) = \lim_{h\to 0} \frac{5(x+h)^2 + 3(x+h) - 7 - (5x^2 + 3x - 7)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{5x^2} + 10xh + 5h^2 + \cancel{3x} + 3h \cancel{-7} \cancel{-5x^2} \cancel{-3x} \cancel{+7}}{\cancel{h}}$$

$$= \lim_{h\to 0} (10x + 5h + 3)$$

$$= 10x + 3.$$

1

7.) (5 pts) Use the precise definition of the limit to prove $\lim_{x\to 3}(5x+2) = 17$

claim: $\lim_{x\to 3} (5x+2) = 17$

1 → proof.

Let $\varepsilon > 0$ be given. choose $\delta = \frac{\varepsilon}{5}$

.5 → If $0 < |x-3| < \frac{\varepsilon}{5}$

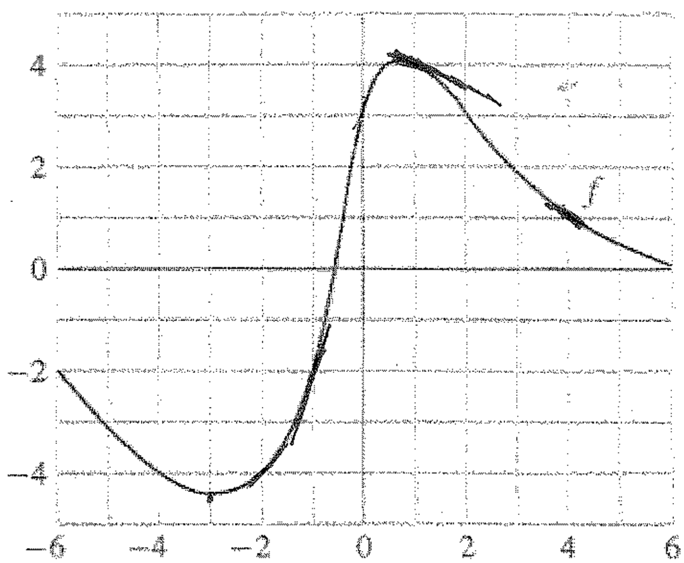
.5 → $\Rightarrow 5|x-3| < \varepsilon$

$\Rightarrow |5x - 15| < \varepsilon$

$\Rightarrow |(5x+2) - 17| < \varepsilon$

Hence $\lim_{x\to 3} (5x+2) = 17.$

8.) Consider the graph of *f*.



a.) Rank $\underbrace{f'(-3)}_{0}$, $\underbrace{f'(-2)}_{1}$, $\underbrace{f'(0)}_{\approx 2}$, $\underbrace{f'(1)}_{-1/2}$, and $\underbrace{f'(\overset{3}{\cancel{4}})}_{-1}$ in increasing order

$f'(\overset{3}{\cancel{4}})$ $\quad f'(1)$ $\quad f'(-3)$ $\quad f'(-2)$ $\quad f'(0)$

smallest ... largest

or $F'(0)$ $\quad f'(-2)$ $\quad f'(-3)$ $\quad f'(1)$ $\quad f'(3)$ for the other version.

b.) Is $f'(-1) > 1$? Justify your answer.

~~no~~ yes, the slope of the tangent is closer to 2

9.) (10 pts) Evaluate $\lim_{x\to\infty} \dfrac{-2x^2 - 2x + 3}{5x^3 + 3x - 7}$

$$= \lim_{x\to\infty} \frac{-2 - \frac{2}{x} + \frac{3}{x^2}}{5x + \frac{3}{x} - \frac{7}{x^2}} \quad \begin{matrix} \to -2 \\ \to \infty \end{matrix}$$

$$= 0$$

10.) State the Squeeze Theorem or the Intermediate Value Theorem (your choice)

Squeeze Thm: If $f(x) \leq g(x) \leq h(x)$ when x is near a (except possibly at a) and the limits

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$$

then $\lim_{x \to a} g(x) = L$.

IVT: Suppose that f is cont. on the closed interval $[a,b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number c in (a,b) s.t. $f(c) = N$.

11.) (5 pts) Evaluate $\lim_{x \to 0} \left(1 + x^2 \cos\left(\frac{1}{x} \right) \right)$

$$-1 \leq \cos \frac{1}{x} \leq 1 \quad (\text{except @ } x = 0)$$

$$\Rightarrow \quad 1 - x^2 \leq 1 + x^2 \cos \frac{1}{x} \leq 1 + x^2$$

and $\lim_{x \to 0} (1 - x^2) = \lim_{x \to 0} (1 + x^2) = 1$

so $\lim_{x \to 0} \left(1 + x^2 \cos \frac{1}{x}\right) = 1$ by the squeeze thm.

12.) (10 pts) Suppose the position of an object moving horizontally after t seconds is given by the function $s(t) = 3t + 4$ where the position is given in meters to the right of the origin.

a.) Find and interpret $s(2)$ $= 10$

after 2 s, the object is 10 m to the right of the origin.

b.) Use the <u>definition of the derivative</u> to find $s'(2)$

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} \frac{3(2+h)+4-10}{h}$$

$$= 3$$

c.) Interpret $s'(2)$

After 2 s, the object is moving at 3 m/s.