4.4: Indeterminate Form & L'Hospital's Rule

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Ex1. $\lim_{x\to 0} \frac{\sin x}{x}$

L'Hospital's Rule bought from Johann Bernoulli

L'Hospital's Rule

Suppose f & g are diff. & $g'(x) \neq 0$ on an open interval that contains a (except possibly at a). Suppose

$\lim_{x\to a} \frac{f(x)}{g(x)}$ approaches the ind. form $\frac{0}{0}$

or $\lim_{x\to a} \frac{f(x)}{g(x)}$ " " " " $\frac{\pm\infty}{\pm\infty}$.

Then $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$ provided the limit on the R.H.S. exists (or is $\pm\infty$).

Ex1 rev: $\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{\cos x}{1} = 1.$

Ex2:

a) $\lim_{x\to\infty} \frac{\ln\sqrt{x}}{x^2}$

b) $\lim_{t\to 1} \frac{t^8 - 1}{t^5 - 1}$

c) $\lim_{x\to 0} \frac{x - \sin x}{x - \tan x} = \lim_{x\to 0} \frac{1 - \cos x}{1 - \sec^2 x} = \lim_{x\to 0} \frac{\sin x}{-2\sec x \cdot \sec x \tan x} = -\frac{1}{2}$

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List the seven Indeterminate Forms on a side board

① $\frac{0}{0}$ ② $\frac{\infty}{\infty}$ — use l'Hospital's Rule

③ $0 \cdot \infty$ ④ $\infty - \infty$ — use algebra to write as a quotient

⑤ 0^0 ⑥ ∞^0 ⑦ 1^∞ — use $e^{\ln x} = x$ to fight the power

ex3: $\lim_{x \to \infty} 3x \tan\left(\frac{1}{x}\right) = \lim_{x \to \infty} \frac{3x}{\frac{1}{\tan\left(\frac{1}{x}\right)}}$

$= \lim_{x \to \infty} \frac{\tan\left(\frac{1}{x}\right)}{\frac{1}{3x}}$

$= \lim_{x \to \infty} \frac{3\tan\left(\frac{1}{x}\right)}{\frac{1}{x}}$ (best)

ex4:

(a) $\lim_{x \to 0} (1-2x)^{1/x}$

(b) $\lim_{x \to \infty} (1+2x)^{3/x}$

(c) $\lim_{x \to \infty} \left(1+\frac{3}{x}\right)^{2x}$