Math 151
Section 3.10: Linear Approximation and Differentials

Video 1: "Finding the Linearization at a Point / Tangent Line Approximation" with URL https://youtu.be/BPSNisGXe7U

Example 1: (start 0:00 and end 4:10) Find the linearization of $f(x)=\frac{1}{\sqrt{7+x}}$ at $x=0$.

Linearization means "find the tangent line".

$x=0 \Rightarrow y=f(0)=\frac{1}{\sqrt{7}}$. [point on line $(0,\frac{1}{\sqrt{7}})$]

$y-\frac{1}{\sqrt{7}}=m(x-0)$

$f'(x) \Rightarrow f'(0)$

$f'(x)=-\frac{1}{2}(7+x)^{-3/2}$

$\Rightarrow f'(0)=\frac{-1}{2\sqrt{7^3}}$ [slope of line]

$\Rightarrow y-\frac{1}{\sqrt{7}}=\frac{-1}{2\sqrt{7^3}}(x-0)$

$=\frac{-1}{14\sqrt{7}}x$

$\Rightarrow y=\frac{-1}{14\sqrt{7}}x+\frac{1}{\sqrt{7}}$

$\Rightarrow L(x)=\frac{1}{\sqrt{7}}\left(1-\frac{1}{14}x\right)$ [often $L(x)$ is used when asked for a linear approximation]

Video 2: "Finding a Linear Approximation (Linearization, Tangent Line Approx), Another Ex 1" with URL https://youtu.be/aaQiNUoZnLE

Video 3: "Finding a Linear Approximation (Linearization, Tangent Line Approx), Another Ex 2" with URL https://youtu.be/Ja2Suuuqjvs

Example 2: (start 0:00 and end 6:53) Find a linear approximation to the given function at the given point.

a.) (Video 2, length 1:59) $f(x) = 2^x$ at $x = 3$

Formula: $y - y_1 = m(x - x_1)$

$f(x) = 2^x$ and $f(3) = 2^3 = 8$

$\Rightarrow f'(x) = 2^x \ln 2$

$\Rightarrow f'(3) = 8 \ln 2$

point $(3, 8)$ and slope $8 \ln 2$

$\Rightarrow y - 8 = 8\ln 2(x - 3)$

$\Rightarrow y - 8 = (8\ln 2)x - 24\ln 2$

$\Rightarrow L(x) = (8\ln 2)x - 24\ln 2 + 8$

b.) (Video 3, length 1:46) $f(x) = \cos x$ at $x = \frac{\pi}{4}$

$f(\pi/4) = \cos \pi/4 = \frac{\sqrt{2}}{2}$ so the point $\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$

$\Rightarrow y - \frac{\sqrt{2}}{2} = m\left(x - \frac{\pi}{4}\right)$

$\uparrow$

$f'(x) = -\sin x$

$\Rightarrow f'(\pi/4) = -\sin \frac{\pi}{4} = -\frac{\sqrt{2}}{2}$ is our slope

$\Rightarrow y - \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$

$\Rightarrow L(x) = -\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}\pi}{8} + \frac{\sqrt{2}}{2}$

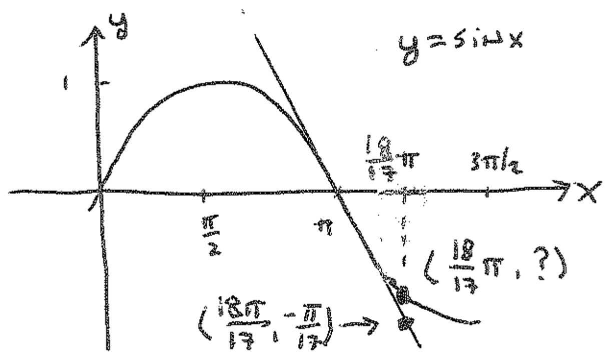
Video 4: "Tangent Line Approximation / Linearization - Ex 1" with URL https://youtu.be/lIV8Zo7IbaY

Example 3: (start 0:00 and end 5:13) Use a linear approximation to approximate the value of each of the following:

a.) (video 4, length 4:48) $\sin\left(\frac{18\pi}{17}\right)$

b.) (no video, solution on my webpage) $\sqrt{15.9}$

(a)



$\frac{18}{17}\pi = 1\frac{1}{17}\pi$

① aquation of the tangent line @ $(\pi, 0)$

② evaluate at $x = \frac{18\pi}{17}$ to approximate. the true value.

part ①: $y - 0 = m(x - \pi)$

$f(x) = \sin x$

$\Rightarrow y = -(x - \pi)$

$\Rightarrow f'(x) = \cos x$

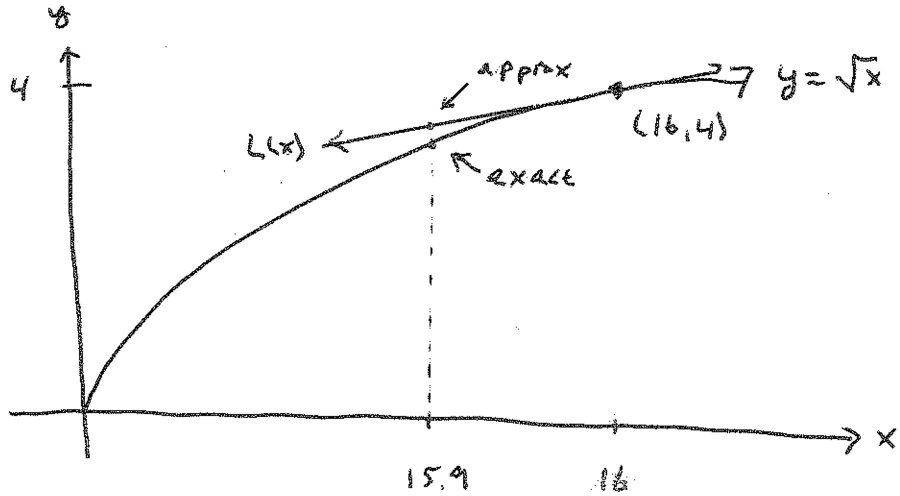
$\Rightarrow L(x) = -x + \pi$

$\Rightarrow f'(\pi) = \cos\pi = -1$

part ②: $\sin\left(\frac{18\pi}{17}\right) \approx -\frac{18\pi}{17} + \pi \leftarrow L\left(\frac{18\pi}{17}\right)$

$= -\frac{\pi}{17}$

(b) approximate $\sqrt{15.9}$



Not to scale.

part ①: $y - 4 = m(x - 16)$

$\Rightarrow y - 4 = \frac{1}{8}(x - 16)$

$\Rightarrow y = \frac{1}{8}x - 2 + 4$

$\Rightarrow L(x) = \frac{1}{8}x + 2$

$f(x) = \sqrt{x}$

$\Rightarrow f'(x) = \frac{1}{2\sqrt{x}}$

$\Rightarrow f'(16) = \frac{1}{8}$

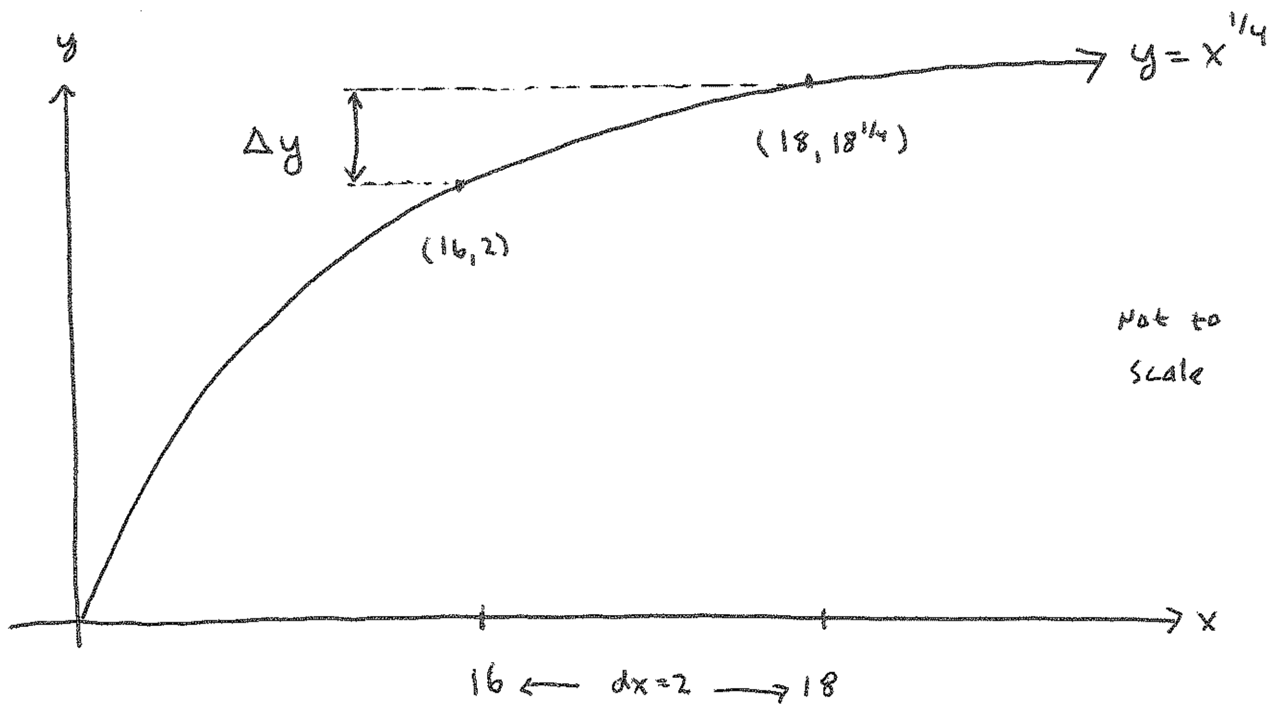
part ②: $\sqrt{15.9} \approx \frac{1}{8}(15.9) + 2 \leftarrow L(15.9)$

$= 3.9875$

Video 5: "Using Differentials" at the URL https://youtu.be/cXIQKIj-NSo

Example 4: (start 0:00 and end 7:37) Use differentials to approximate the value of $18^{1/4} = \sqrt[4]{18}$

Vocabulary: $dy =$ differential (the approximate change) with the formula is $dy = f'(x)dx$
$\Delta y =$ the true change in y



$f(x) = x^{1/4}$

$dy = \frac{1}{4}x^{-3/4}dx$

16 2

$\Rightarrow dy = \frac{1}{4} \cdot \frac{1}{2^3} \cdot 2 = \frac{2}{32} = \frac{1}{16}$

We want $18^{1/4} = 2 + \Delta y$

$\approx 2 + dy$

So $18^{1/4} \approx 2 + \frac{1}{16}$

$= \frac{33}{16}$