Math 151

3.5: Implicit Differentiation Workalong

Based on the Khan Academy videos

Video 1: "Implicit Differentiation" with URL https://youtu.be/sL6MC-IKOrw

Concept 1: (start 0:00 and end 2:00) Implicit vs. explicit equations

Example of an ______ equation: $y = x^2 + 2x + 3$

Example of an ______ equation: $x^2 + y^2 = 100$

Example 1: (start 2:01 and end 11:25) Find the derivative $\frac{dy}{dx}$ of the implicitly defined equation $x^2 + y^2 = 100$

Aside on the chain rule

$$g = y^2$$

$$\frac{d}{dx}[g] = \frac{d}{dx}[y^2]$$

Example 2: (start 11:26 and end 15:56) Find the derivative $\frac{dy}{dx}$ of the implicitly defined equation $y = x^x$

YOU MAY SKIP THIS EXAMPLE.

THIS REQUIRES LOGS WHICH WE DON'T INTRODUCE UNTIL LATER

Video 2: "Implicit Differentiation (part 2)" with URL https://youtu.be/PUsMyhds5S4

Example 3: (length 10:44) Find the derivative $\frac{dy}{dx}$ of the equation $y^3 - xy^2 + \cos(xy) = 2$.

Aside
$$g = xy$$

$$\frac{d}{dg} \left[\cos(g) \right] = -\sin(g)$$

Video 3: "More implicit differentiation" with URL https://youtu.be/hrg1hCzg3W0

Example 4: (0:00 - 6:45) Find the derivative $\frac{dy}{dx}$ of the equation $x^2 - 24xy + 16y^2 - 400x - 300y = 0$.

Aside
Note:
$$\frac{dy}{dx} = y'$$

Example 5: (6:46 - 11:29) Find the derivative $\frac{dy}{dx}$ of the equation $(x-y)^2 = 8(y-6)$.