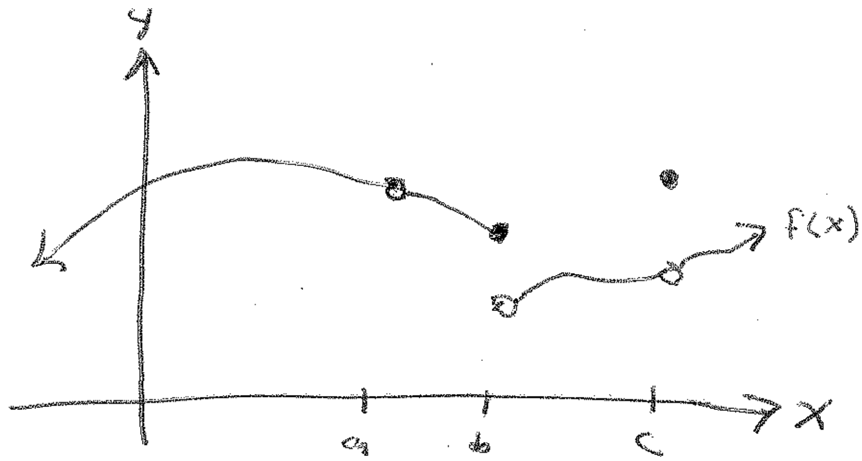
2.5: Continuity

Dfn: A fct f is cont. @ $x=a$ if $\lim_{x\to a} f(x) = f(a)$

3 ways it fails:



ex1: Explain why (not) $f(x) = \begin{cases} \frac{2x^2-5x-3}{x-3}, & x \neq 3 \\ 6, & x = 3 \end{cases}$

is cont. @ $x=3$.

Dfn: cont. from the left & right.

Dfn: A fct f is cont. on an interval if it is cont. @ every number in the interval. (If f is defined only on one side of an endpt of the interval, we understand cont. @ the endpt to mean cont. from the right or left).

<u>Thm:</u> If f & g are cont. @ a and c is a const. then the following are also cont. @ a:

$f+g$ $\quad$ $f-g$ $\quad$ $c \cdot f$ $\quad$ $f \cdot g$ $\quad$ * $\frac{f}{g}$, $g(a) \neq 0$.

□ <u>proof of *.</u>

Since f & g are both cont!

$\lim_{x \to a} f(x) = f(a)$ & $\lim_{x \to a} g(x) = g(a)$. Assuming $g(a) \neq 0$,

we have $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ (2.3, rule 5)

$$= \frac{f(a)}{g(a)}$$

$$= \left(\frac{f}{g}\right)(a) \quad , g(a) \neq 0$$

Hence $\frac{f}{g}$ is cont. when $g(a) \neq 0$ ■

We can now expand our direct sub. prop. from sec. 2.3. The following types of fcts are cont. at every number in their domain.

polynomials $\quad$ trig $\quad$ exp fcts

rat. fcts $\quad$ inv. trig $\quad$ log fcts.

root fcts.

Note: If a 1-1 fct is cont., then so is its inverse. (why?)

ex2: where is $f(x) = \begin{cases} x+1, & x \le 1 \\ 3 + \frac{1}{x-2}, & 1 < x < 3 \\ \sqrt{x-3}, & x \ge 3 \end{cases}$ cont.?

Thm: If f is cont. @ b & $\lim_{x \to a} g(x) = g(a) = b$

then $\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right) = f(b)$

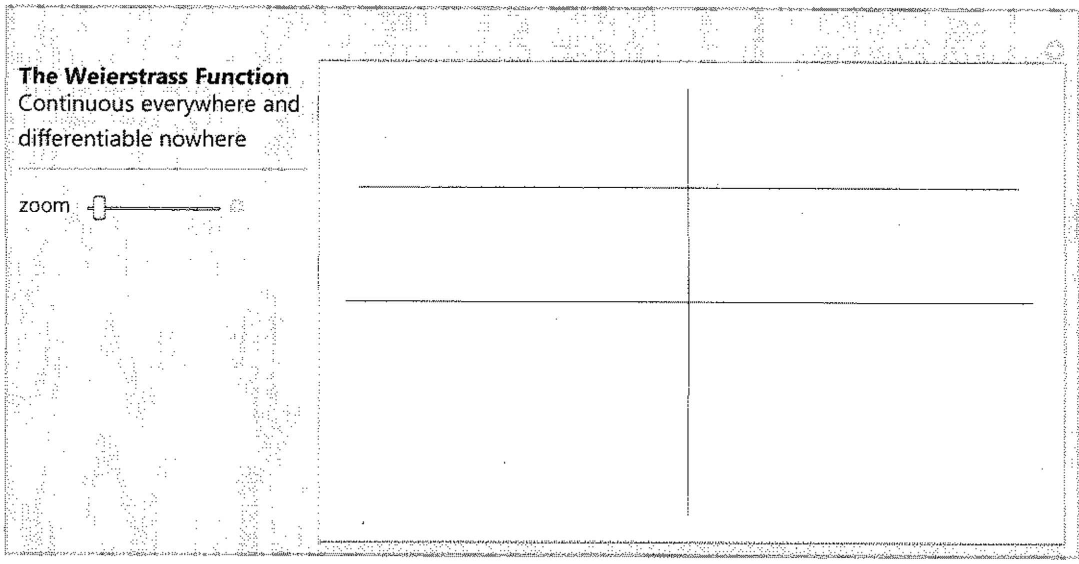
similarly, continuity carries thru compositions.

Thm: The IVT

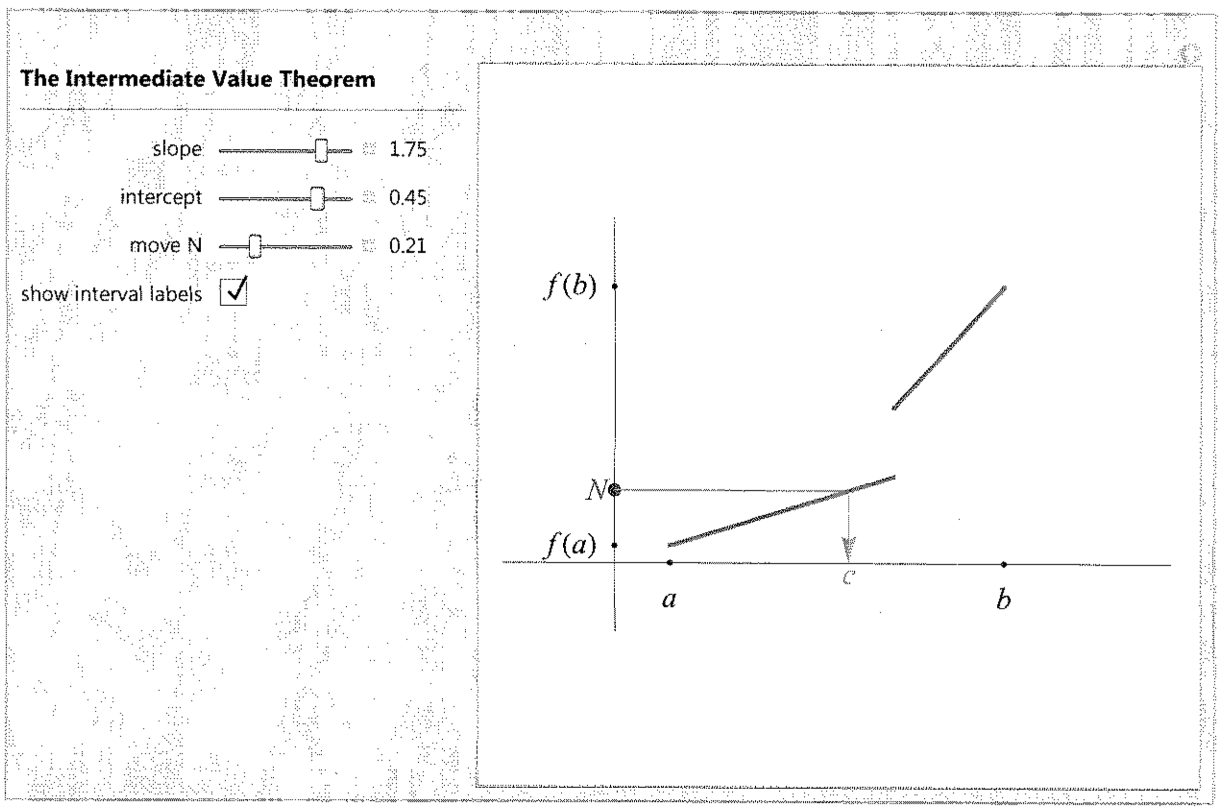
Suppose f is cont. on $[a,b]$ and N is between $f(a)$ and $f(b)$ where $f(a) \neq f(b)$. Then $\exists\, c \in (a,b)$ s.t. $f(c) = N$.

show the pic. (mathematica).

Out[4]=


The Weierstrass Function
Continuous everywhere and differentiable nowhere
zoom

Out[5]=


The Intermediate Value Theorem
slope
1.75
intercept
0.45
move N
0.21
show interval labels
f(b)
N
f(a)
a
c
b