2.2: The Limit of a Function

Begin by exploring $f(x) = \dfrac{x-3}{x^2-x-6}$ for x values near $x = 3$.

Q: can we evaluate $f(3)$?

Dfn: We write $\lim_{x \to a} f(x) = L$

and say, "the limit of $f(x)$, as x approaches a, equals L"
if we can make the values of $f(x)$ arbitrarily close to L by taking x sufficiently close to a, but not equal to a.

Look @ the mathematica examples.
Helpful examples: $\lim_{x\to 0} \frac{1}{x}$ and $\lim_{x\to 0} \frac{1}{x^2}$. (see dfn & note)
Three good examples from the text (numerical & graphical)

(a) $\lim_{x \to 0} \dfrac{\sqrt{x^2+16}-4}{x^2}$

(b) $\lim_{x \to 0} \dfrac{\sin x}{x}$

(c) $\lim_{x \to 0} \sin\left(\dfrac{\pi}{x}\right)$

If we construct all these tables using $x = \pm 1, \pm .1, \pm .01, \ldots$ the last example (c) has the added benefit of giving a false limit.

Introduce one sided limits & their connection to limits.

Introduce infinite limits.

- Graphical examples
- Numeric examples.

Dfn. Let a fct f be defined on both sides of a, except possibly @ a itself. Then $\lim_{x \to a} f(x) = \infty$ means that the values of f can be made arbitrarily large by taking x sufficiently close to a.

Note: This doesn't mean we regard ∞ as a number or that the limit exists. Rather it gives info about how the limit DNE.