3.1: Derivatives of polys & exps.

Derive the pattern of the power rule for pos. int.

<u>claim</u>: $\frac{d}{dx} x^N = N x^{N-1}$ for $N \in \mathbb{Z}^+$

□ <u>proof</u>

Suppose $f(x) = x^N$

$$\Rightarrow f'(a) = \lim_{x \to a} \frac{x^N - a^N}{x - a}$$

$$= \lim_{x \to a} \frac{(x-a)(x^{N-1} + x^{N-2}a + \dots + xa^{N-2} + a^{N+1})}{x-a}$$

$$= \lim_{x \to a} (x^{N-1} + x^{N-2}a + \dots + xa^{N-2} + a^{N\mp 1})$$

$$= N a^{N-1}$$

and so $f'(x) = N x^{N-1}$ ■

It also works for $N \in \mathbb{R}$.

The normal line.

Derivatives of $c \cdot f(x)$ & $(f+g)(x)$.

<u>claim</u>: If f & g are both differentiable, then

$$\frac{d}{dx}(f(x) - g(x)) = \frac{d}{dx} f(x) - \frac{d}{dx} g(x).$$

□ <u>proof</u>.

$$\frac{d}{dx}(f(x) - g(x)) = \lim_{h \to 0} \frac{(f(x+h) - g(x+h)) - (f(x) - g(x))}{h}$$

$$= \lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h} - \frac{g(x+h)-g(x)}{h}\right]$$

$$= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} - \lim_{h\to 0}\frac{g(x+h)-g(x)}{h}$$

$$= \frac{d}{dx}f(x) - \frac{d}{dx}g(x) \quad \blacksquare$$

ex: For what values of x does the graph of $f(x) = x^3 + 3x^2 + x + 3$ have a horizontal tangent

finding velocity & acceleration.

working w/ exp. fcts.

If $f(x) = a^x$, find $f'(0) = \lim_{h\to 0}\frac{a^{0+h} - a^0}{h}$

$$= \lim_{h\to 0}\frac{a^h - 1}{h}$$

find $f'(x) = \lim_{h\to 0}\frac{a^{x+h} - a^x}{h}$

$$= \lim_{h\to 0}\frac{a^x a^h - a^x}{h}$$

$$= \lim_{h\to 0}\frac{a^x(a^h - 1)}{h}$$

$$= a^x \lim_{h\to 0}\frac{a^h - 1}{h}$$

$$= f'(0)\cdot a^x$$

The RoC of the exponential is proportional to the fct. itself.

Notice that if $f(x) = a^x$, then $f'(0)$ depends upon the value of a.

Q: what value of a results in $f'(0) = 1$.

that is, $\frac{d}{dx} a^x = a^x$?

<u>Dfn:</u> e

e is the number s.t. $\lim\limits_{h \to 0} \frac{e^h - 1}{h} = 1$