Graphing Quadratics (8.6 and 8.7) **Math 098**

Graphing quadratic functions requires a strong understanding of the "toolkit" function $f(x) = x^2$

Table	Graph

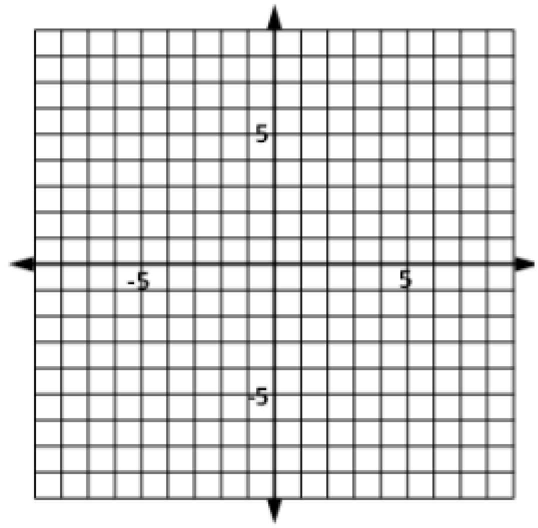
With that toolkit knowledge, we can graph the transformed "toolkit" quadratic $f(x) = a(x-h)^2 + k$

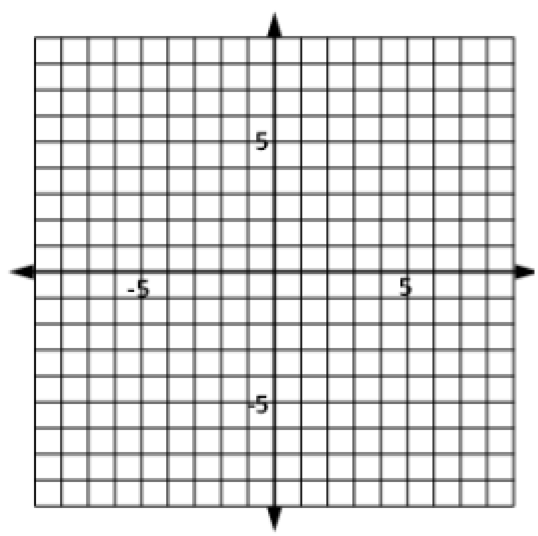
Let's explore each of the parameters: *a, h,* and *k.*

- $f(x) = \mathbf{a}(x-h)^2 + k$
  - If $a > 0$, the quadratic is ________________ or, more precisely, ________________
  - If $a < 0$, the quadratic is ________________ or, more precisely, ________________
- $f(x) = a\left(x - \mathbf{h}\right)^2 + k$
  - Shift the quadratic ________________ or ________________ in the ________________ ___________________ of the sign you see.
- $f(x) = a(x-h)^2 + \mathbf{k}$
  - Shift the quadratic ________________ or ________________ in the ________________ ___________________ as the sign you see.

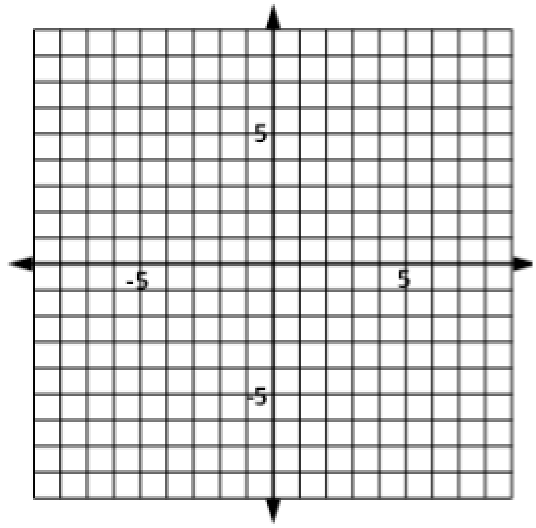
Note: A more rigorous development of these concepts (including the reasons why they work) can be found in section 8.6 of the text.

Example 1: Graph accurately

a.) $f\left(x\right)=\frac{1}{2}\left(x+4\right)^{2}-5$	
	
Vertex:	Domain:
y-intercept:	Range:

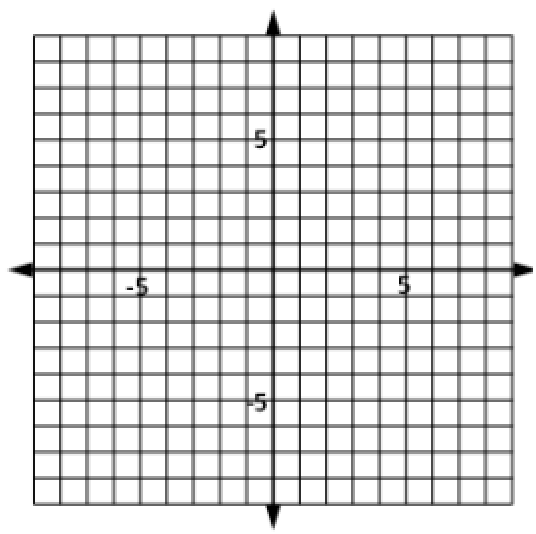
b.) $g\left(x\right)=-3\left(x-2\right)^{2}+3$	
	
Vertex:	Domain:
y-intercept:	Range:

Example 2: Graph $f(x)=x^2-8x+9$ by completing the square

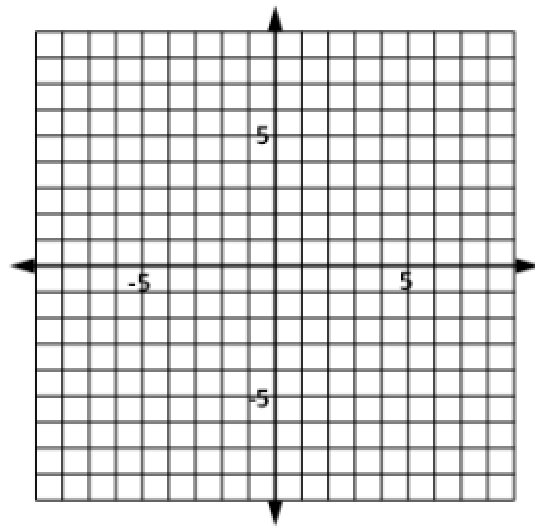


Vertex:	Domain:
y-intercept:	Range:

Example 3: Graph $g(x)=2x^2+4x+6$ by completing the square



Vertex:	Domain:
y-intercept:	Range:

Example 4: Graph $h(x) = -\frac{1}{2}x^2 + 3x - \frac{1}{2}$ by completing the square	
	
Vertex:	Domain:
y-intercept:	Range:

<u>Find the Formula</u>: Find the vertex of the general quadratic $f(x) = ax^2 + bx + c$ by completing the square.

Method: The vertex of a parabola

a.) The vertex of the parabola given by $f(x)=ax^2+bx+c$ is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

b.) The longer version of the formula is: $\left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right)$

c.) The *x*-coordinate of the vertex is $-\frac{b}{2a}$. The equation of the axis of symmetry is $x=-\frac{b}{2a}$. The second coordinate of the vertex is most commonly found by computing $f\left(-\frac{b}{2a}\right)$.

Example 5: Find the vertex of $g(x)=2x^2+5x-1$. Check with your graphing calculator.

Example 6: Consider $f(x) = 4x^2 - 12x + 3$. Find the vertex, all intercepts, the min/max value, and the range.

Example 7: Consider $g(x) = -18.8x^2 + 7.92x + 6.18$. Find the vertex, all intercepts, the min/max value, and the range.

<u>Summary</u>: The graph of a quadratic equation given by $f(x) = ax^2 + bx + c$ or $f(x) = a(x-h)^2 + k$

a.) The graph is a parabola

b.) The vertex is (h,k) or $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

c.) The axis of symmetry is $x = h$ or $x = -\frac{b}{2a}$

d.) The *y*-intercept of the graph is $(0,c)$

e.) The *x*-intercepts can be found by solving $ax^2 + bx + c = 0$

  a. If $b^2 - 4ac > 0$, there are two real *x*-intercepts

  b. If $b^2 - 4ac = 0$, there is one *x*-intercept

  c. If $b^2 - 4ac < 0$, there are no real *x*-intercepts (although there are two complex zeros)

f.) The domain of the function is $(-\infty, \infty)$

g.) If $a > 0$:

  a. The graph opens upward

  b. The function has a minimum value of *k* at (h,k)

  c. The range of the function is $[k, \infty)$

h.) If $a < 0$:

  a. The graph opens downward

  b. The function has a maximum value of *k* at (h,k)

  c. The range of the function is $(-\infty, k]$