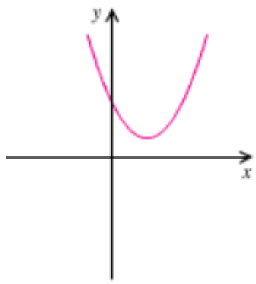
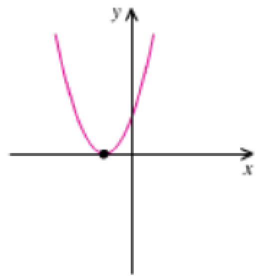
Quadratic Equations (8.1) **Math 098**

As seen in Math 091 and earlier in Math 098, the graphs of quadratic equations are parabolic in shape.

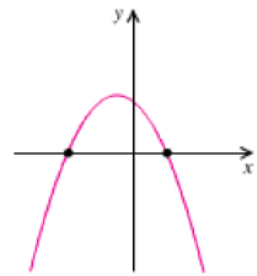
When solving quadratic equations, there are three cases:



No *x*-intercepts
No real-valued roots/zeros



One *x*-intercept
One real-valued root/zero



Two *x*-intercepts
Two real-valued roots/zeros

Example 1: Solve $6x^2 = x + 12$

Example 2: Solve $x^2 = 49$

Intuitively, how might we solve the last example $x^2 = 49$?

Method: The principle of square roots

a.) For any real number *k*, if $x^2 = k$, then $x = \sqrt{k}$ or $x = -\sqrt{k}$.

Sometimes we write this using the notation: ______________________

Example 3: Solve

a.) $4x^2 = 20$

b.) $-7x^2 + 6 = 0$

c.) $9x^2 + 10 = 0$

Example 4: Let $f(x) = (x+3)^2$, find all values of *x* such that $f(x) = 6$. Find algebraic and graphical solutions.

Example 5: Solve $x^2 - 10x + 25 = 3$

Review:

a.) $x^2 + 8x + 16 = (x \_\_\_\_\_\_\_\_\_\_)^2$

b.) $x^2 - 10x + 25 = (x \_\_\_\_\_\_\_\_\_\_)^2$

c.) $x^2 - 7x + \frac{49}{4} = (x \_\_\_\_\_\_\_\_\_\_)^2$

This leads us to a slick way to solve quadratic equations via completing the square.

Example 6: Solve $x^2 + 6x - 2 = 0$

Example 7: What number should be used to "complete the square"?

a.) $x^2 + 12x + \_\_\_\_\_\_\_\_\_\_ = (x \_\_\_\_\_\_\_\_\_\_)^2$

b.) $x^2 - 3x + \_\_\_\_\_\_\_\_\_\_ = (x \_\_\_\_\_\_\_\_\_\_)^2$

c.) $x^2 - \frac{4}{3}x + \_\_\_\_\_\_\_\_\_\_ = (x \_\_\_\_\_\_\_\_\_\_)^2$

Example 8: Solve $x^2 - 10x - 3 = 0$ by completing the square.

<u>Method</u>: To solve a quadratic equation in *x* by completing the square

a.) Isolate the terms with variables on one side of the equation, and arrange them in descending order.
b.) Divide both sides by the coefficient of x^2 if that coefficient is not 1.
c.) Complete the square by taking half of the coefficient of *x* and adding its square to both sides.
d.) Express the trinomial as the square of a binomial (factor the trinomial) and simplify the other side.
e.) Use the principle of square roots (find the square roots of both sides).
f.) Solve for *x* by adding or subtracting on both sides.

Example 9: Solve $4x^2 + 3x - 20 = 0$

Example 10: Find the *x*-intercepts of $y = 2x^2 - 5x - 3$

Formula: The compound interest formula

a.) If any amount of money P is invested at interest rate r, compounded annually, then in t years, it will grow to the amount A given by $A = P(1+r)^t$ where r is written in decimal notation.

Example 11: Find the interest rate if \$6,250 is invested and grows to \$7,290 in 2 years.

Example 12: The formula $s = 16t^2$ is used to approximate the distance s in feet, that an object falls freely from rest in t seconds. Ireland's Cliffs of Moher are 702 ft tall. How long will it take a stone to fall from the top? Round to the nearest tenth of a second.