Rational Exponents (7.2) **Math 098**

Definition: $a^{1/n} = \sqrt[n]{a}$. When *a* is nonnegative, *n* can be any natural number greater than 1. When *a* is negative, *n* must be odd.

Example 1: Write in radical notation and simplify.

a.) $x^{1/2}$

b.) $(-27)^{1/3}$

c.) $\left(365^{12}\right)^{1/2}$

Example 2: Write with exponential notation.

a.) $\sqrt[4]{7ab}$

b.) $\sqrt[5]{\dfrac{3x}{7y}}$

Example 3: Graph $f(x) = \sqrt[4]{3x-2}$ on your calculator.

<u>Definition</u>: (Positive rational exponents) For any natural numbers *m* and *n* ($n \neq 0$) and any real number *a* for which $\sqrt[n]{a}$ exists, we have that $a^{m/n}$ means $\left(\sqrt[n]{a}\right)^m$ or $\sqrt[n]{a^m}$

Example 4: Write in radical notation and simplify

a.) $8^{2/3}$

b.) $36^{3/2}$

<u>Definition</u>: (Negative rational exponents) For any rational number m/n and any nonzero real number *a* for which $a^{m/n}$ exists, we have that $a^{-m/n}$ means $\dfrac{1}{a^{m/n}}$.

Example 5: Write with positive exponents and simplify if possible.

a.) $49^{-1/2}$

b.) $(-27)^{-2/3}$

c.) $5a^{-3/2}b^{4/3}$

d.) $\left(\dfrac{x}{y}\right)^{-3/5}$

<u>Definition</u>: (Laws of exponents) For any real numbers *a* and *b* and any rational exponents *m* and *n* for which a^m, a^n, and b^m are defined:

1.) $a^m \cdot a^n = a^{m+n}$ In multiplying, add exponents if the bases are the same.

2.) $\dfrac{a^m}{a^n} = a^{m-n}$ In dividing, subtract exponents if the bases are the same. Assume $a \neq 0$.

3.) $\left(a^m\right)^n = a^{m \cdot n}$ To raise a power to a power, multiply the exponents.

4.) $(ab)^m = a^m b^m$ To raise a product to a power, raise each factor to the power and multiply.

Example 6: Simplify (answers should have positive exponents)

a.) $5^{3/7} \cdot 5^{1/7}$

b.) $\dfrac{a^{1/6}}{a^{1/2}}$

c.) $\left(\pi^{3/4}\right)^{2/3}$

d.) $\left(r^{-1/4}\, b^{3/7}\right)^{1/3}$

<u>Method</u>: To simplify radical expressions

1.) Convert radical expressions to exponential expressions.
2.) Use arithmetic and the laws of exponents to simplify.
3.) Convert back to radical notation as needed.

Example 7: Simplify

a.) $\sqrt[4]{s^{12}}$

b.) $\left(\sqrt[5]{x^2 y}\right)^{20}$

c.) $\sqrt[8]{(3y)^4}$

d.) $\sqrt[3]{\sqrt{r}}$