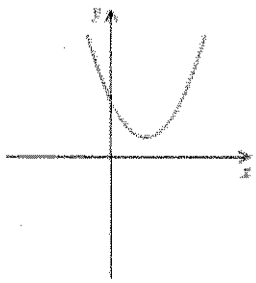
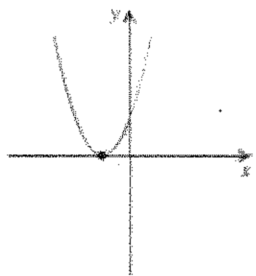
Quadratic Equations (8.1) **Math 098**

As seen in Math 091 and earlier in Math 098, the graphs of quadratic equations are parabolic in shape.

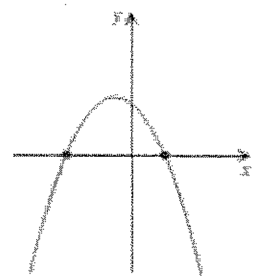
When solving quadratic equations, there are three cases:



No x-intercepts
No real-valued roots/zeros



One x-intercept
One real-valued root/zero



Two x-intercepts
Two real-valued roots/zeros

Example 1: Solve $6x^2 = x + 12$

$\Rightarrow 6x^2 - x - 12 = 0$

$\Rightarrow (2x-3)(3x+4) = 0$

$\Rightarrow x = -\frac{4}{3}$ or $x = \frac{3}{2}$

Example 2: Solve $x^2 = 49$

$\Rightarrow x^2 - 49 = 0$

$\Rightarrow (x+7)(x-7) = 0$

$\Rightarrow x = \pm 7.$

Intuitively, how might we solve the last example $x^2 = 49$?

$x^2 = 49$

$\Rightarrow \sqrt{x^2} = \sqrt{49}$

$\Rightarrow |x| = 7$

$\Rightarrow x = \pm 7$

when you $\sqrt{\ }$ both sides of an eqn., you pick up "$\pm$"

Method: The principle of square roots

a.) For any real number k, if $x^2 = k$, then $x = \sqrt{k}$ or $x = -\sqrt{k}$.

Sometimes we write this using the notation: $x = \pm\sqrt{k}$

Example 3: Solve

a.) $4x^2 = 20$ $\Rightarrow$ $x^2 = 5$

$\Rightarrow$ $x = \pm\sqrt{5}$

b.) $-7x^2 + 6 = 0$ $\Rightarrow$ $-7x^2 = -6$

$\Rightarrow$ $x^2 = \frac{6}{7}$

$\Rightarrow$ $x = \pm\sqrt{\frac{6}{7}}$

$i = \sqrt{-1}$

$i^2 = -1$

negative under $\sqrt{\ }$ comes out as i.

c.) $9x^2 + 10 = 0$ $\Rightarrow$ $9x^2 = -10$

$\Rightarrow$ $x^2 = -\frac{10}{9}$

$\Rightarrow$ $x = \pm\sqrt{-\frac{10}{9}}$

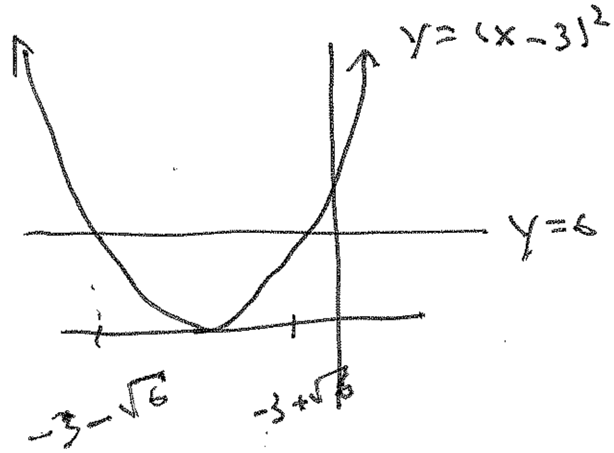
$\Rightarrow$ $x = \pm i\frac{\sqrt{10}}{3}$

Example 4: Let $f(x) = (x+3)^2$, find all values of x such that $f(x) = 6$. Find algebraic and graphical solutions.

Solve $(x+3)^2 = 6$

$\Rightarrow$ $x + 3 = \pm\sqrt{6}$

$\Rightarrow$ $x = -3 \pm \sqrt{6}$



Example 5: Solve $x^2 - 10x + 25 = 3$

$\Rightarrow (x-5)^2 = 3$

$\Rightarrow x - 5 = \pm\sqrt{3}$

$\Rightarrow x = 5 \pm \sqrt{3}$

Review:

a.) $x^2 + 8x + 16 = (x \underline{\;+4\;})^2$ $\qquad \frac{8}{2} = 4$

b.) $x^2 - 10x + 25 = (x \underline{\;-5\;})^2$ $\qquad \frac{-10}{2} = -5$

c.) $x^2 - 7x + \frac{49}{4} = (x \underline{\;-\frac{7}{2}\;})^2$ $\qquad \frac{-7}{2} = -\frac{7}{2}$

This leads us to a slick way to solve quadratic equations via <u>completing the square.</u>

Example 6: Solve $x^2 + 6x - 2 = 0$

$\Rightarrow x^2 + 6x = 2$

$\Rightarrow x^2 + 6x + 9 = 2 + 9$

$\Rightarrow (x+3)^2 = 11$

$\Rightarrow x + 3 = \pm\sqrt{11}$

$\Rightarrow x = -3 \pm \sqrt{11}$

Example 7: What number should be used to "complete the square"?

a.) $x^2 + 12x + \underline{\;36\;} = (x \underline{\;+6\;})^2$ $\qquad \left(\frac{12}{2}\right)^2 = 36$

b.) $x^2 - 3x + \underline{\;\frac{9}{4}\;} = (x \underline{\;-\frac{3}{2}\;})^2$ $\qquad \left(\frac{-3}{2}\right)^2 = \frac{9}{4}$

c.) $x^2 - \frac{4}{3}x + \underline{\;\frac{4}{9}\;} = (x \underline{\;-\frac{2}{3}\;})^2$ $\qquad \left(\frac{-\frac{4}{3}}{2}\right)^2 = \left(\frac{-2}{3}\right)^2 = \frac{4}{9}$

Example 8: Solve $x^2 - 10x - 3 = 0$ by completing the square.

$\left(\frac{-10}{2}\right)^2 = 25$

$\Rightarrow x^2 - 10x = 3$

$\Rightarrow x^2 - 10x + 25 = 3 + 25$

$\Rightarrow (x-5)^2 = 28$

$\Rightarrow x - 5 = \pm\sqrt{28}$

$\Rightarrow x = 5 \pm \sqrt{28}$

$\Rightarrow x = 5 \pm 2\sqrt{7}$

<u>Method</u>: To solve a quadratic equation in *x* by completing the square

a.) Isolate the terms with variables on one side of the equation, and arrange them in descending order.

b.) Divide both sides by the coefficient of x^2 if that coefficient is not 1.

c.) Complete the square by taking half of the coefficient of *x* and adding its square to both sides.

d.) Express the trinomial as the square of a binomial (factor the trinomial) and simplify the other side.

e.) Use the principle of square roots (find the square roots of both sides).

f.) Solve for *x* ~~b~~ by adding or subtracting on both sides.

Example 9: Solve $4x^2 + 3x - 20 = 0$

$\left(\frac{\frac{3}{4}}{2}\right)^2 = \left(\frac{3}{8}\right)^2 = \frac{9}{64}$

$\Rightarrow 4x^2 + 3x = 20$

$\Rightarrow x^2 + \frac{3}{4}x = 5$

$\Rightarrow x^2 + \frac{3}{4}x + \frac{9}{64} = 5 + \frac{9}{64}$

$\Rightarrow \left(x + \frac{3}{8}\right)^2 = \frac{329}{64}$

$\Rightarrow x + \frac{3}{8} = \pm\sqrt{\frac{329}{64}}$

$\Rightarrow x = -\frac{3}{8} \pm \frac{\sqrt{329}}{8}$

Example 10: Find the <u>x-intercepts</u> of $y = 2x^2 - 5x - 3$

$= (2x+1)(x-3)$

$y = 0$

$\left(\frac{-\frac{5}{2}}{2}\right)^2 = \left(-\frac{5}{4}\right)^2 = \frac{25}{16}$

solve $2x^2 - 5x - 3 = 0$

$\Rightarrow 2x^2 - 5x = 3$

$\Rightarrow x^2 - \frac{5}{2}x = \frac{3}{2}$

$\Rightarrow x^2 - \frac{5}{2}x + \frac{25}{16} = \frac{3}{2} + \frac{25}{16}$

$\Rightarrow \left(x - \frac{5}{4}\right)^2 = \frac{49}{16}$

$\Rightarrow x - \frac{5}{4} = \pm\sqrt{\frac{49}{16}}$

$\Rightarrow x = \frac{5}{4} \pm \frac{7}{4}$

$\Rightarrow x = 3$ or $x = -\frac{1}{2}$

Formula: The compound interest formula

a.) If any amount of money P is invested at interest rate r, compounded annually, then in t years, it will grow to the amount A given by $A = P(1+r)^t$ where r is written in decimal notation.

Example 11: Find the interest rate if \$6,250 is invested and grows to \$7,290 in 2 years.

$A = 7290$

$P = 6250$

r unknown

$t = 2$

solve $7290 = 6250(1+r)^2$

$\Rightarrow \frac{7290}{6250} = (1+r)^2$

$\Rightarrow \pm\sqrt{1.1664} = 1+r$

$\Rightarrow r = 1.08 - 1$

$\Rightarrow \quad = 0.08$

The interest rate is 8%.

Example 12: The formula $s = 16t^2$ is used to approximate the distance s in feet, that an object falls freely from rest in t seconds. Ireland's Cliffs of Moher are 702 ft tall. How long will it take a stone to fall from the top? Round to the nearest tenth of a second.

$s = 702$

t unknown

solve $702 = 16t^2$

$\Rightarrow \frac{702}{16} = t^2$

$\Rightarrow t = \pm\sqrt{\frac{702}{16}}$

$\approx \pm 6.6$ s

It will take 6.6 seconds for the stone to reach the bottom.