Complex Numbers (7.8) **Math 098**

Definition: The number *i*

i is the unique number for which $i=\sqrt{-1}$ and $i^2=-1$

We can now define the root $\sqrt{-a}=\sqrt{-1}\sqrt{a}=i\sqrt{a}$ provided *a* is non-negative.

Warning: $i \neq$ -1

Example 1: Express in terms of *i*.

a.) $\sqrt{-15}$

$=\sqrt{15}\,i$

$=i\sqrt{15}$

b.) $\sqrt{-9}$

$=i\sqrt{9}$

$=i3$

$=3i$

c.) $-\sqrt{-50}$

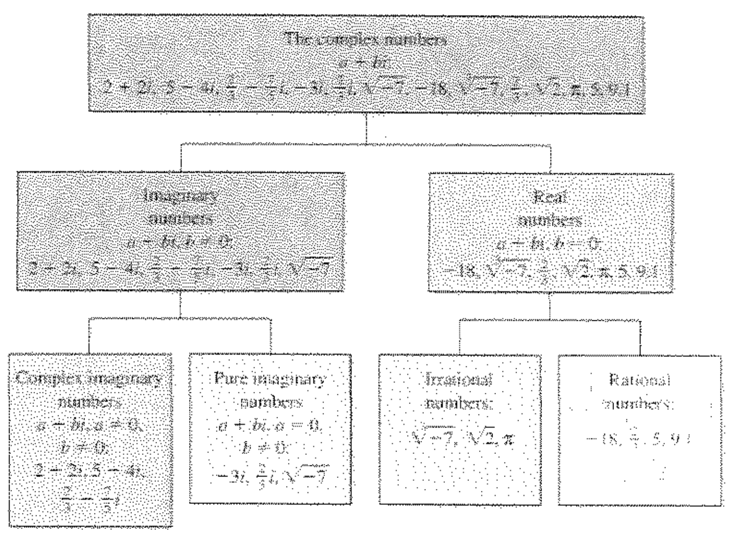
$=-i\sqrt{50}$

$=-5i\sqrt{2}$

$=-i5\sqrt{2}$

Definition: Imaginary numbers
~~An *imaginary number* is a number that can be written in the form $a+bi$, where *a* and *b* are real numbers and $b\neq 0$.~~

These have many real world applications in engineering and the physical sciences. Some applications include: control theory, improper integrals, fluid dynamics, dynamic equations, electromagnetism and electrical engineering, signal analysis, quantum mechanics, relativity, geometry, fractals, algebraic number theory, and analytic number theory



Definition: Complex numbers
A *complex number* is a number that can be written in the form $a+bi$, where *a* and *b* are real numbers. Note that both *a* and *b* can be 0.

Example 2: Add or subtract

a.) $(4-5i)+(2+3i)$

$= 4-5i+2+3i$

$= 6-2i$

b.) $(3-i)-(5-2i)$

$= 3-i-5+2i$

$= -2+i$

Warning: $\sqrt{-3}\cdot\sqrt{-3} = i\sqrt{3}\cdot i\sqrt{3}$

$= i^2(\sqrt{3})^2$

$= -3$

vs. $\sqrt{-3}\cdot\sqrt{-3} = \sqrt{(-3)\cdot(-3)}$

$= \sqrt{9}$

$= 3$

EPIC FAIL: $\sqrt{-3}$ is NOT REAL

Example 3: Multiply and simplify. Write you answers in the standard $a+bi$ form

a.) $\sqrt{-9}\cdot\sqrt{-36}$

$= i\sqrt{9}\, i\sqrt{36}$

$= i^2\cdot 3\cdot 6$

$= -18$

b.) $\sqrt{-6}\cdot\sqrt{-10}$

$= i^2\sqrt{6}\sqrt{10}$

$= -\sqrt{60}$

$= -2\sqrt{15}$

c.) $-2i\cdot 7i$

$= -14i^2$

$= +14$

d.) $3i(4-7i)$

$= 12i-21i^2$

$= 12i+21$

$= 21+12i$

e.) $(2-3i)(4+5i)$

$= 8+10i-12i-15i^2$

$= 8-2i+15$

$= 23-2i$

f.) $(3-5i)^2 = 9-30i+25i^2$

$= 9-30i-25$

$= -16-30i$

Definition: Conjugate of a complex number

The *conjugate* of a complex number $a+bi$ is $a-bi$ and the conjugate of $a-bi$ is $a+bi$.

Example 4: Find and multiply by the conjugate

a.) $-2+5i$

conjugate: $-2-5i$

$(-2+5i)(-2-5i)$

$= 4+\cancel{10i}-\cancel{10i}-25i^2$

$= 4+25$

$= 29$

b.) $3-7i$

conjugate: $3+7i$

$(3-7i)(3+7i)$

$= 9+\cancel{21i}-\cancel{21i}-49i^2$

$= 9+49$

$= 58$

c.) $5i$

conjugate: $-5i$

$5i(-5i)$

$= -25i^2$

$= 25$

Method: When dividing by complex numbers, we multiply by the complex conjugate as a special one in a manner similar to how we rationalize the denominator.

Example 5: Divide. Write your answers in the form $a+bi$

a.) $\frac{4}{2-3i} \cdot \frac{2+3i}{2+3i}$

$= \frac{8+12i}{4-\cancel{6i}+\cancel{6i}-9i^2}$

$= \frac{8+12i}{13}$

$= \frac{8}{13}+\frac{12}{13}i$

both ok.

$i=\sqrt{-1}$ and $i^2=-1$

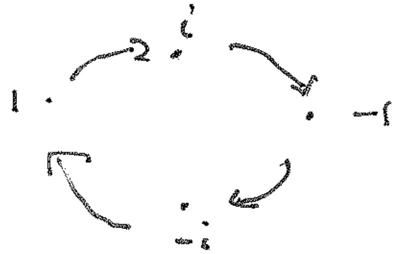
b.) $\frac{2+7i}{5i} \cdot \frac{-5i}{-5i}$

$= \frac{-10i-35i^2}{-25i^2}$

$= \frac{-10i+35}{25}$

$= \frac{-2i+7}{5}$

$= \frac{7}{5}-\frac{2}{5}i$



Explore powers of i

$i =$ i

$i^2 =$ -1

$i^3 =$ $-i$

$i^4 =$ $-i^2 = +1$

$i^5 =$ i

$i^6 =$ -1

$i^7 =$ $-i$

$i^8 =$ $+1$

Divide powers by 4

Remainder	Result
0	1
1	i
2	-1
3	$-i$

Example 6: Simplify

a.) $i^{28} = 1$

$$\begin{array}{r}7\text{ R}0\\4\overline{)28}\end{array}$$

b.) $i^{46} = -1$

$$\begin{array}{r}11\text{ R}2\\4\overline{)46}\end{array}$$

c.) $i^{33} = i$

$$\begin{array}{r}8\text{ R}1\\4\overline{)33}\end{array}$$

d.) $i^{75} = -i$

$$\begin{array}{r}18\text{ R}3\\4\overline{)75}\end{array}$$

You can also work with complex numbers on the graphing calculator ...

use i on the calculator.