Radical Equations (7.6) **Math 098**

Definition: A radical equation contains radical in the equation..

Method: The principle of powers

If $a = b$, then $a^n = b^n$ for any exponent n.

Notice the "if-then" relationship here. For example, examine $x = 3 \Rightarrow x^2 = 9$

Warning: We must check for extraneous solutions

Example 1: Solve $\sqrt{x} - 5 = 4$ (solve algebraically and graphically)

$\Rightarrow \sqrt{x} = 9$

$\Rightarrow (\sqrt{x})^2 = 9^2$

$\Rightarrow x = 81$

check ✓

Method: To solve an equation with a radical term

1.) Isolate the radical term on one side of the equation.
2.) Use the principle of powers and solve the resulting equation.
3.) Check any possible solution in the original equation.

Example 2: Solve $\sqrt{x}+5=2$

$\Rightarrow \sqrt{x} = -3$

$\Rightarrow (\sqrt{x})^2 = (-3)^2$

$\Rightarrow \cancel{x = 9}$

check.

No solution.

Example 3: $\sqrt{x-2}-7=-4$

$\Rightarrow \sqrt{x-2} = 3$

$\Rightarrow (\sqrt{x-2})^2 = 3^2$

$\Rightarrow x-2 = 9$

$\Rightarrow x = 11$

check. ✓

Example 4: $x=\sqrt{x-1}+3$

$\Rightarrow (x-3)^2 = (\sqrt{x-1})^2$

$\Rightarrow x^2-6x+9 = x-1$ $\leftarrow$ Quadratic Eqn.

$\Rightarrow x^2-7x+10=0$

$\Rightarrow (x-5)(x-2)=0$

$\Rightarrow x = \overset{\checkmark}{5}$ or $\cancel{x=2}$.

check.

Method: Solve an equation with two or more radical terms

1.) Isolate one of the radical terms.
2.) Use the principle of powers.
3.) If a radical remains, perform steps (1.) and (2.) again.
4.) Solve the resulting equation.
5.) Check possible solutions in the original equation.

Example 5: Solve

a.) $\sqrt{x+2}+\sqrt{3x+4}=2$

$\Rightarrow \sqrt{x+2} = 2 - \sqrt{3x+4}$

$\Rightarrow (\sqrt{x+2})^2 = (2-\sqrt{3x+4})^2$

$\Rightarrow x+2 = 4 - 4\sqrt{3x+4} + 3x+4$

$\Rightarrow \frac{-2x-6}{-2} = \frac{-4\sqrt{3x+4}}{-2}$ divide both sides by -2.

* $\Rightarrow x+3 = 2\sqrt{3x+4}$

$\Rightarrow \frac{x+3}{2} = \sqrt{3x+4}$

$\Rightarrow \left(\frac{x+3}{2}\right)^2 = 3x+4$

$\Rightarrow \frac{x^2+6x+9}{4} = 3x+4$

$\Rightarrow x^2+6x+9 = 12x+16$

$\Rightarrow x^2-6x-7=0$

$\Rightarrow (x-7)(x+1)=0$

$\Rightarrow \cancel{x=7}$ or $x=-1$

check.

* Alternate Method.

$x+3 = 2\sqrt{3x+4}$

$\Rightarrow (x+3)^2 = (2\sqrt{3x+4})^2$

$\Rightarrow x^2+6x+9 = 4(3x+4)$

$\Rightarrow x^2+6x+9 = 12x+16$

b.) $\sqrt{6x+7}-\sqrt{3x+3}=1$

$\Rightarrow (\sqrt{6x+7})^2 = (1+\sqrt{3x+3})^2$

$\Rightarrow 6x+7 = 1 + 2\cdot 1\cdot\sqrt{3x+3} + 3x+3$

$\Rightarrow (3x+3)^2 = (2\sqrt{3x+3})^2$

$\Rightarrow 9x^2+18x+9 = 4(3x+3)$

$= 12x+12$

$\Rightarrow \frac{9x^2+6x-3}{3} = \frac{0}{3}$

$\Rightarrow 3x^2+2x-1=0$

$\Rightarrow (3x-1)(x+1)=0$

$\Rightarrow x=\frac{1}{3}$ or $x=-1$

check.

Example 6: For the given functions, find the values of *t*.

a.) If $f(t)=\sqrt{t-2}-\sqrt{4t+1}$, solve $f(t)=-3$

$\Rightarrow$ solve $-3=\sqrt{t-2}-\sqrt{4t+1}$

$\Rightarrow \sqrt{4t+1}-3=\sqrt{t-2}$

$\Rightarrow (\sqrt{4t+1}-3)^2=(\sqrt{t-2})^2$

$\Rightarrow 4t+1-6\sqrt{4t+1}+9=t-2$

$\Rightarrow 3t+12=6\sqrt{4t+1}$

$\Rightarrow \dfrac{3t+12}{3}=\dfrac{6\sqrt{4t+1}}{3}$

$\Rightarrow t+4=2\sqrt{4t+1}$

$\Rightarrow (t+4)^2=(2\sqrt{4t+1})^2$

$\Rightarrow t^2+8t+16=4(4t+1)$

$=16t+4$

$\Rightarrow t^2-8t+12=0$

$\Rightarrow (t-6)(t-2)=0$

$\Rightarrow t=6$ or $t=2$.

check.

$t=6$: $-3=\sqrt{4}-\sqrt{25}$ ✓

$t=2$: $-3=\sqrt{0}-\sqrt{9}$ ✓

b.) If $g(t)=\sqrt{t}+\sqrt{t-9}$, solve $g(t)=1$

$\sqrt{t}+\sqrt{t-9}=1$

$\Rightarrow \sqrt{t-9}=1-\sqrt{t}$

$\Rightarrow (\sqrt{t-9})^2=(1-\sqrt{t})^2$

$\Rightarrow \cancel{t}-9=1-2\sqrt{t}+\cancel{t}$

$\Rightarrow -10=-2\sqrt{t}$

$\Rightarrow 5=\sqrt{t}$

$\Rightarrow \cancel{t=25}$

check

$\sqrt{25}+\sqrt{16}\neq 1$

"No solution"