Multiplying Radical Expressions (7.3) **Math 098**

Example 1: Examine

a.) $\sqrt{9 \cdot 16}$ vs. $\sqrt{9} \cdot \sqrt{16}$

$\sqrt{9 \cdot 16} = \sqrt{144} = 12$

$\sqrt{9} \cdot \sqrt{16} = 3 \cdot 4 = 12$

b.) $\sqrt{9+16}$ vs. $\sqrt{9} + \sqrt{16}$

$\neq \quad \sqrt{9+16} = \sqrt{25} = 5$

$\sqrt{9} + \sqrt{16} = 3 + 4 = 7$

<u>Definition</u>: (The product rule for radicals) For any real numbers $\sqrt[n]{a}$ and $\sqrt[n]{b}$, we have $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$
That is, the product of two *n*th roots is the *n*th root of the product of the two radicands.

Example 2: Multiply

a.) $\sqrt{5} \cdot \sqrt{6} = \sqrt{5 \cdot 6}$

$= \sqrt{30}$

b.) $\sqrt{x-4} \cdot \sqrt{x+4}$

$= \sqrt{(x-4)(x+4)}$

$= \sqrt{x^2 - 16}$

c.) $\sqrt{2} \cdot \sqrt{8} = \sqrt{16}$

$= 4$

d.) $\sqrt[3]{9} \cdot \sqrt[3]{3}$

$= \sqrt[3]{27}$

$= 3$

Method: Using the product rule to simplify

$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ where $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are both real numbers

Example 3: Simplify $\sqrt{50}$ (The Jail Story)

$\sqrt{2 \cdot 25} = 5\sqrt{2}$

Method: To simplify a radical expression with index *n* by factoring

1.) Express the radicand as a product in which one factor is the largest perfect *n*th power possible.
2.) Take the *n*th root of each factor.
3.) Simplification is complete when no radicand has a factor that is a perfect *n*th power.

Example 4: Simplify

a.) $\sqrt{27} = \sqrt{3 \cdot 9}$

$= 3\sqrt{3}$

b.) $\sqrt[3]{40} = \sqrt[3]{8 \cdot 5}$

$= 2\sqrt[3]{5}$

c.) $\sqrt[4]{162} = \sqrt[4]{2 \cdot 3^4}$

$= 3\sqrt[4]{2}$

d.) $\sqrt{169p^4r^6}$

$= \sqrt{169} \cdot \sqrt{p^4} \cdot \sqrt{r^6}$

$= 13p^2r^3$

$32 = 2 \cdot 16$

e.) $\sqrt{81y^5} = 9y^2\sqrt{y}$

f.) $\sqrt{32xy^2} = 4y\sqrt{2x}$

$32 = 2^5$

g.) $\sqrt[4]{32z^7}$

$= \sqrt[4]{2^5 z^7}$

$= 2z\sqrt[4]{2z^3}$

$24 = 8 \cdot 3$

h.) $\sqrt[3]{24a^9b^4}$

$= 2a^3b\sqrt[3]{3b}$

Example 5: You try to simplify $\sqrt[3]{108x^{14}y^{27}z^{34}}$

$= \sqrt[3]{2^2 \cdot 3^3 x^{14} y^{27} z^{34}}$

$= 3x^4y^9z^{11}\sqrt[3]{4x^2z}$

Example 6: Simplify $f(x) = \sqrt{2x^2 - 8x + 8}$

$= \sqrt{2(x^2 - 4x + 4)}$

$= \sqrt{2(x-2)^2}$

$= |x-2|\sqrt{2}$

Example 7: Multiply and simplify

a.) $\sqrt{10}\cdot\sqrt{14} = \sqrt{2\cdot 5\cdot 2\cdot 7} = 2\sqrt{35}$

2 5 2 7

b.) $\sqrt[3]{4}\cdot\sqrt[3]{20} = \sqrt[3]{\underbrace{2\cdot 2}_{4}\cdot\underbrace{2\cdot 2\cdot 5}_{20}} = 2\sqrt[3]{10}$

c.) $\sqrt[4]{4a^3b^5}\cdot\sqrt[4]{20a^2b^7} = \sqrt[4]{2\cdot 2\, a^3 b^5 \cdot 2\cdot 2 \cdot 5\, a^2 b^7}$

$= 2\sqrt[4]{5a^5b^{12}}$

$= 2ab^3\sqrt[4]{5a}$