Polynomial Equations and Factoring (5.3) **Math 098**

$Y_1 \quad Y_2$

Example 1: Consider $x^2 = -5x$

a.) Using the graphing calculator, solve using the *intersect method*.

$y1 = x^2$

$y2 = -5x$

Find the pts where the graphs intersect

$x = 0$ OR $x = -5$

2nd → calc → intersect

b.) Using the graphing calculator, solve using the *zero method*.

$x^2 = -5x$

$\Rightarrow x^2 + 5x = 0$

$y1 = x^2 + 5x$

$x = 0$ OR $x = -5$

2nd → calc → zero

expressions
fcts

Vocabulary: Zeros and Roots: The *x*-values for which a function $f(x)$ is 0 are called the *zeros* of the function. The *x*-values for which an equation such as $f(x) = 0$ is true are called the *roots* of the equation.

equation

Example 2: Find the zeros of the function $f(x) = x^3 - 2x^2 - 3x$ using the graphing calculator.

Solve $f(x) = 0$

$y1 = x^3 - 2x^2 - 3x$

$x = -1, 0, 3$

Here is a very important obvious fact. <u>The principle of zero products</u>: For any real numbers *a* and *b*, $ab = 0$ if and only if $a = 0$ or $b = 0$.

When a polynomial is written as a product, we say it is factored.

The zeros of a polynomial function are zeros described by the factors of the polynomial.

a b

Example 3: Solve $(x-2)(x+5)=0$

$\Rightarrow x-2=0$ OR $x+5=0$

$\Rightarrow x=2$ OR $x=-5$

Example 4: Given $f(x)=x(2x+5)$, find the zeros of the function.

Solve $f(x)=0$

a b

$\Rightarrow 0 = x(2x+5)$

$\Rightarrow a=0$ OR $b=0$

$\Rightarrow x=0$ OR $2x+5=0$

$2x=-5$

$\Rightarrow x=0$ OR $x=-\frac{5}{2}$

To factor, an expression means to write it as a product.

To factor out the greatest common factor (GCF) we will do reverse distribution.

Example 5: Factor out the greatest common factor (GCF)

a.) $6x^3 - 24$ $= 6(x^3 - 4)$

GCF = 6

b.) $12r^2s^3 - 9r^5s^6 + 15r^3s^2$ $= 3r^2s^2(4s - 3r^3s^4 + 5r)$

GCF = $3r^2s^2$

c.) $-5x^2 + 10x - 25$ $= -5(x^2 - 2x + 5)$

GCF = 5 OR -5

d.) $-4x^4 + 6x^3 - 2x^2$ $= -2x^2(2x^2 - 3x + 1)$

GCF = $2x^2$ OR $-2x^2$

Example 6: Factor by grouping

a.) $(x-2)(x^2-3)+(x-2)(5-3x^2)$

$= (x-2)\left((x^2-3)+(5-3x^2)\right)$

$= (x-2)(x^2-3+5-3x^2)$

$= (x-2)(2-2x^2)$

b.) b^3-b^2+2b-2

$= (b^3-b^2) + (2b-2)$

$= b^2\cancel{(b-1)} + 2\cancel{(b-1)}$

$= (b-1)(b^2+2)$

c.) $t^3+6t^2-2t-12$

$= (t^3+6t^2) - (2t+12)$

$= t^2\cancel{(t+6)} - 2\cancel{(t+6)}$

$= (t+6)(t^2-2)$

d.) $ax-bx+by-ay$

$= (ax-bx)+(by-ay)$

$= x(a-b)+y(b-a)$

$= x(a-b)-y(a-b)$

$= (a-b)(x-y)$

Example 7: Solve $8x^2 = 40x$

$8x^2 - 40x = 0$

$\Rightarrow 8x(x-5) = 0$

$\Rightarrow 8x = 0$ OR $x-5=0$

$\Rightarrow x = 0$ OR $x = 5$

Summary: <u>To use the principle of zero products</u>

1.) Write an equivalent equation with 0 on one side, using the additions principle.
2.) Factor the nonzero side of the equation.
3.) Set each factor that is not a constant equal to 0.
4.) Solve the resulting equations.