Chapter 7: Rational Exponents and Radicals

- Understand basic roots including how to evaluate them by hand and using a calculator.
- Find the domain and range of a radical function (the latter part using the graph).
- Understand the relationship between rational exponents and roots.
- Note: You can check your work on the calculator, but must justify answers algebraically (unless otherwise noted).

Review questions:

Example 1: Consider $f(x)=\frac{x-5}{x+1}$. Find all values of a for which $f(a)=\frac{1}{5}$.

Solve $\frac{a-5}{a+1}=\frac{1}{5}$

$\Rightarrow 5(a-5)=1(a+1)$

$\Rightarrow 5a-25=a+1$

$\Rightarrow 4a=26$

$\Rightarrow a=\frac{26}{4}$

$\Rightarrow a=\frac{13}{2}$

Example 2: Simplify $\frac{\frac{1}{y}+3}{\frac{1}{y}-4}$

$=\frac{\frac{1+3y}{y}}{\frac{1-4y}{y}}$

$=\frac{1+3y}{1-4y}$.

Example 3: Simplify $\frac{\frac{1}{x^2-3x+2}+\frac{1}{x^2-4}}{\frac{1}{x^2+4x+4}+\frac{1}{x^2-4}}$

$=\frac{\frac{1}{(x-2)(x-1)}+\frac{1}{(x+2)(x-2)}}{\frac{1}{(x+2)^2}+\frac{1}{(x+2)(x-2)}}$

$=\frac{\frac{(x+2)+(x-1)}{(x+2)(x-1)(x-2)}}{\frac{(x-2)+(x+2)}{(x+2)^2(x-2)}}$

$=\frac{\frac{2x+1}{(x+2)(x-2)(x-1)}\cdot\frac{x+2}{x+2}}{\frac{2x}{(x+2)^2(x-2)}\cdot\frac{x-1}{x-1}}$

$=\frac{(2x+1)(x+2)}{2x(x-1)}$

Example 4: Write with positive exponents: $3^{-5/2}a^3b^{-7/3}$ $=\frac{a^3}{3^{5/2}b^{7/3}}$

Example 5: Simplify $\frac{x+6}{5x+10}-\frac{x-2}{4x+8}$

$$= \frac{x+6}{5(x+2)} - \frac{x-2}{4(x+2)}$$

$$= \frac{4(x+6)-5(x-2)}{20(x+2)}$$

$$= \frac{-x+34}{20(x+2)}$$

Example 6: Simplify $\frac{x}{x^2+14x+48}-\frac{6}{x^2+10x+24}$. For what values is the expression undefined?

$$= \frac{x}{(x+6)(x+8)} - \frac{6}{(x+6)(x+4)}$$

$$= \frac{x(x+4)-6(x+8)}{(x+4)(x+6)(x+8)} \leftarrow x^2+4x-6x-48 \quad x^2-2x-48$$

$$= \frac{(x-8)\cancel{(x+6)}}{(x+4)\cancel{(x+6)}(x+8)}$$

$$= \frac{x-8}{(x+4)(x+8)}$$

This is undefined when $x = 4, 6, 8$.

Example 7: Consider the function $f(x)=\frac{x^2+x-12}{x^2-8x+15} = \frac{(x+4)\cancel{(x-3)}}{(x-5)\cancel{(x-3)}} = \frac{x+4}{x-5}$

$x \neq 3, 5$

a.) Find the domain (express your answer in interval notation).

$(-\infty, 3) \cup (3, 5) \cup (5, \infty)$

b.) Give the equation(s) of the vertical asymptote(s)

$x = 5$

c.) Are there any holes? Justify your answer.

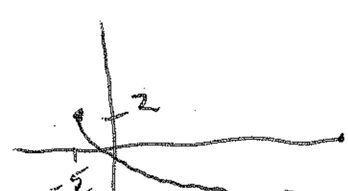
$x = 3$ (the cancelled term/lost info).

Example 8: Determine the domain of $f(x)=2-\sqrt{3x+5}$. Use the graph to find the range.

Domain: $3x+5 \geq 0$

$\Rightarrow 3x \geq -5$

$\Rightarrow x \geq -\frac{5}{3}$



Range: $y \leq 2$

Example 9: Solve $\frac{3}{x-3}+\frac{2}{x+1}=\frac{4x}{x^2-2x-3}$

$$\Rightarrow \frac{3}{x-3}+\frac{2}{x+1}=\frac{4x}{(x+1)(x-3)}$$

$$\Rightarrow 3(x+1)+2(x-3)=4x$$

$$\Rightarrow 3x+3+2x-6=4x$$

$$\Rightarrow x \cancel{=} 3 \quad \text{check}$$

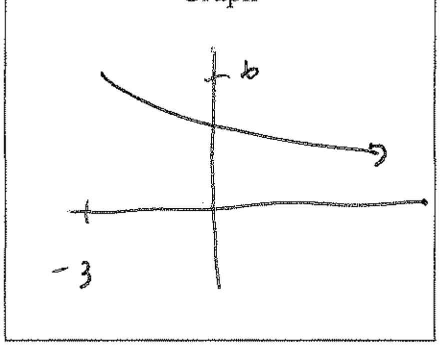
No solution.

Example 10: Algebraically find the domain of $f(x)=6-\sqrt{x+3}$ and then use the graph to find the range. Sketch the graph in the box to justify your range.

Domain: $x+3 \geq 0$

$x \geq -3$

Graph



Domain: $x \geq -3$

Range: $y \leq 6$

Example 11: Write an equivalent expression to $6^{-\frac{2}{3}}x^5y^{-\frac{5}{4}}$ with positive exponents.

$$= \frac{x^5}{6^{2/3}\,y^{5/4}}$$

Example 12: If $g(x)=\sqrt{x^3+9}$, evaluate $g(-2)$, $g(-3)$, and $g(3)$

$g(-2) = \sqrt{1} = 1$

$g(-3) = \sqrt{-27+9} = \sqrt{-18}$ Not a real number.

$g(3) = \sqrt{27+9} = \sqrt{36} = 6$