

- Factoring the difference of squares, perfect squares, and the sum/difference of cubes.
- *Solving Equations*
 - Put equations in standard form
 - If the polynomial can be factored, then the zeros of each factor are the solution to the equation.
 - If the polynomial can't be factored, graph it and look for its zeros.

Chapter 6: Rational Expressions, Equations, and Functions

- Multiply straight across
- Divide by inverting and multiplying
- Expressions are undefined when the denominator is zero.

Review questions:

Example 1: Consider the function

$$f(x) = \frac{x^2 + x - 12}{x^2 - 8x + 15} = \frac{(x+4)(x-3)}{(x-5)(x+3)}$$

- a.) Find the domain (express your answer in interval notation).

$$(-\infty, 3) \cup (3, 5) \cup (5, \infty)$$

- b.) Give the equation(s) of the vertical asymptote(s)

$$x = 5$$

- c.) Are there any holes? Justify your answer.

$$x = 3 \quad (\text{the lost information})$$

Example 2: Factor $8x^2 - 8y^2$

$$\begin{aligned} &= 8(x^2 - y^2) \\ &= 8(x+y)(x-y) \end{aligned}$$

Example 3: Solve $4x^2 - 9 = 0$

$$\Rightarrow (2x+3)(2x-3) = 0$$

$$\Rightarrow x = \pm \frac{3}{2}$$

Factor

Example 3: $6a^2 + 7a - 10$
 $= (6a - 5)(a + 2)$

$$\begin{array}{r|rrrr} 3 & 6 & 7 & -10 & \\ 2 & 1 & 5 & -2 & \\ \hline & 6 & 7 & -10 & \\ & 1 & 5 & -2 & \end{array}$$

$$\begin{array}{r|rrrr} 6 & 6 & 7 & -10 & \\ 1 & 1 & 5 & -2 & \\ \hline & 6 & 7 & -10 & \\ & 1 & 5 & -2 & \end{array}$$

Factor

Example 4: $12m^2 + 19m - 18$
 $= (4m + 9)(3m - 2)$

$$\begin{array}{r|rrrr} 4 & 12 & 19 & -18 & \\ 3 & 4 & 6 & -3 & \\ \hline & 12 & 19 & -18 & \\ & 4 & 6 & -3 & \end{array}$$

Example 5: Factor $ab^3 + 125a$

$$= a(b^3 + 125)$$

$$= a(b + 5)(b^2 - 5b + 25)$$

Example 6: Factor $27x^3 - 8$

$$= (3x - 2)(9x^2 + 6x + 4)$$

Example 7: Simplify $\frac{c^3 + 8}{c^5 - 4c^3} \cdot \frac{c^6 - 4c^5 + 4c^2}{c^2 - 2c + 4}$

$$= \frac{(c+2)(c^2 - 2c + 4) \cdot c^2(c^4 - 4c^2 + 4)}{c^3(c^2 - 4)(c^2 - 2c + 4)}$$

$$= \frac{(c+2)(c^4 - 4c^2 + 4)}{c(c-2)(c+2)}$$

$$= \frac{c^4 - 4c^2 + 4}{c(c-2)}$$

Example 8: Simplify $\frac{7x - 49}{x^3 - 9x} \div \frac{x^2 - 2x - 35}{x^3 - 3x^2}$

$$= \frac{7(x-7)}{x(x^2 - 9)} \cdot \frac{x^2(x-3)}{(x-7)(x+5)}$$

$$= \frac{7x^2(x-3)}{x(x+3)(x-3)(x+5)}$$

$$= \frac{7x}{(x+3)(x+5)}$$