Chapter 6: Rational Expressions, Equations, and Functions

- Multiply straight across
- Divide by inverting and multiplying (KFC)
- To add/subtract, make sure you have a common denominator
- To simplify complex rational expressions
  - Method 1: multiply by a special one formed from the LCD of the full expression.
  - Method 2: combine terms until left with the quotient of two rational expressions. The invert and multiply.
- Expressions are undefined when the denominator is zero.
- To solve rational equations, multiply both sides of the equation by the LCD. Make sure to check for extraneous solutions.

Review questions (from the online practice test):

Example 1: Consider $f(x)=\dfrac{x-5}{x+1}$. Find all values of a for which $f(a)=\dfrac{1}{5}$.

solve $\dfrac{a-5}{a+1} = \dfrac{1}{5}$ LCD $= 5(a+1)$

$\Rightarrow 5(a-5) = 1\cdot(a+1)$

$\Rightarrow 5a - 25 = a + 1$

$\Rightarrow 4a = 26$

$\Rightarrow a = \dfrac{13}{2}$ //.

Example 2: $\dfrac{c^3+8}{c^5-4c^3}\cdot\dfrac{c^6-4c^5+4c^2}{c^2-2c+4}$ $= \dfrac{(c+2)\cancel{(c^2-2c+4)}}{c^3(c^2-4)}\cdot\dfrac{\cancel{c^2}(c^4-4c^3+4)}{\cancel{c^2-2c+4}}$

$= \dfrac{\cancel{(c+2)}(c^4-4c^3+4)}{c\,\cancel{(c+2)}(c-2)}$

$= \dfrac{c^4-4c^3+4}{c(c-2)}$ //

Example 3: Simplify $\dfrac{\dfrac{1}{y}+3}{\dfrac{1}{y}-4}$ $= \dfrac{\dfrac{1+3y}{y}}{\dfrac{1-4y}{y}}$

$= \dfrac{1+3y}{1-4y}$ //

Example 4: Simplify $\dfrac{\dfrac{1}{x^2-3x+2}+\dfrac{1}{x^2-4}}{\dfrac{1}{x^2+4x+4}+\dfrac{1}{x^2-4}}$

$$= \dfrac{\dfrac{1}{(x-2)(x-1)}+\dfrac{1}{(x+2)(x-2)}}{\dfrac{1}{(x+2)^2}+\dfrac{1}{(x+2)(x-2)}}$$

$$= \dfrac{\dfrac{(x+2)+(x-1)}{(x-1)(x+2)(x-2)}}{\dfrac{(x-2)+x+2}{(x-2)(x+2)^2}} \quad \begin{matrix} \leftarrow 2x+1 \\ \\ \leftarrow 2x \end{matrix}$$

$$= \frac{(2x+1)\cancel{(x-2)}(x+2)^{\cancel{2}}}{2x(x-1)\cancel{(x+2)}\cancel{(x-2)}} = \frac{(2x+1)(x+2)}{2x(x-1)} \quad //.$$

Example 5: $\dfrac{x+6}{5x+10}-\dfrac{x-2}{4x+8}$

$$= \frac{x+6}{5(x+2)} - \frac{x-2}{4(x+2)} \qquad LCD = 20(x+2)$$

$$= \frac{4(x+6)-5(x-2)}{20(x+2)}$$

$$= \frac{4x+24-5x+10}{20(x+2)}$$

$$= \frac{34-x}{20(x+2)} \quad //.$$

Example 6: Simplify $\dfrac{x}{x^2+14x+48}-\dfrac{6}{x^2+10x+24}$. For what <u>values</u> is the expression undefined?

$$= \frac{x}{(x+6)(x+8)} - \frac{6}{(x+6)(x+4)}$$

$$LCD = (x+4)(x+6)(x+8)$$

$$= \frac{x(x+4)-6(x+8)}{(x+4)(x+6)(x+8)} \leftarrow x^2 \underbrace{+4x-6x}_{-2x} -48$$

$$= \frac{(x-8)\cancel{(x+6)}}{(x+4)\cancel{(x+6)}(x+8)}$$

$$= \frac{x-8}{(x+4)(x+8)} \quad //.$$

The expression is undefind for $x=-4,-6,-8$

Example 7: Simplify $\frac{7x-49}{x^3-9x} \div \frac{x^2-2x-35}{x^3-3x^2}$ $= \frac{7(\cancel{x-7})}{\cancel{x}(x^2-9)} \cdot \frac{x^{\cancel{2}}(x-3)}{(\cancel{x-7})(x+5)}$

$$= \frac{7x(\cancel{x-3})}{(x+3)(\cancel{x-3})(x+5)}$$

$$= \frac{7x}{(x+3)(x+5)} \; //.$$

Example 8: Consider the function $f(x) = \frac{x^2+x-12}{x^2-8x+15}$. $= \frac{(x+4)(\cancel{x-3})}{(x-5)(\cancel{x-3})} = \frac{x+4}{x-5}$

a.) Find the domain (express your answer in interval notation).

all real numbers except $x = 3$ and $x = 5$. In interval notation this is: $(-\infty, 3) \cup (3, 5) \cup (5, \infty)$

b.) Give the equation(s) of the vertical asymptote(s)

$x = +5$

c.) Are there any holes? Justify your answer.

$x = 3$ since f is undefined at this value yet it is not problematic when f is simplified.

Example 9: Solve $\frac{3}{x-3} + \frac{2}{x+1} = \frac{4x}{x^2-2x-3}$

$\Rightarrow \frac{3}{x-3} + \frac{2}{x+1} = \frac{4x}{(x-3)(x+1)}$ LCD $= (x-3)(x+1)$

$\Rightarrow 3(x+1) + 2(x-3) = 4x$

$\Rightarrow 5x - 3 = 4x$

$\Rightarrow x \cancel{=} 3$ $\leftarrow$ extraneous soln.

check

No solution//.