14.3 : Partial Derivatives.

The R.G. covered the basics.

Picture partial derivatives (Mathematica).

ex1 Consider $f(x,y) = xy^2 + x^3$

(a) find $f_x(x,y) = y^2 + 3x^2$

we also write this as $\frac{\partial f}{\partial x}$ or f_x

(b) Find & interpret $f_y(1,2) = 4$

Note: x is fixed... y varies.

pts $(1,y) \longmapsto f(1,y) = y^2+1$

(see mathematica)

ex2: Find $\frac{\partial z}{\partial x}$ of $\cos(xyz) = 3x + 2y + z$

implicit diff $\Rightarrow -\sin(xyz)\cdot\left(yz + xy\frac{\partial z}{\partial x}\right) = 3 + \frac{\partial z}{\partial x}$

Solve for $\frac{\partial z}{\partial x}$ $\Rightarrow -yz\sin(xyz) - xy\sin(xyz)\frac{\partial z}{\partial x} - 3 = \frac{\partial z}{\partial x}$

$$\Rightarrow \frac{\partial z}{\partial x} = -\frac{yz\sin(xyz) + 3}{1 + xy\sin(xyz)}.$$

As w/ derivatives of fcts of 1 variable, we may be able to take higher order derivatives. However, we must choose which variable to differentiate WRT @ each step.

<u>ex 3:</u> $f(x,y) = x^3 e^{5y} + y\sin(2x)$

(a) f_x

f_y

(b) f_{xx}

1st 2nd

f_{xy}

(c) $\dfrac{\partial^2 f}{\partial y^2}$

$\dfrac{\partial^2 f}{\partial x\, \partial y}$

2nd. 1st

This notation makes more sense when thought of as the differential operator.

fct: $f(x,y)$

1st ∂: $\dfrac{\partial}{\partial x}\big(f(x,y)\big) = \dfrac{\partial f}{\partial x}$

2nd ∂: $\dfrac{\partial}{\partial y}\left(\dfrac{\partial f}{\partial x}\right) = \dfrac{\partial^2 f}{\partial y\, \partial x}$

Notes: pure v. mixed partials.

mixed partials are sometimes equal.

<u>Clairaut's Thm</u>: Suppose f is defined on a disk D that contains the pt (a,b). If the fcts f_{xy} & f_{yx} are both cont on D, then $f_{xy}(a,b) = f_{yx}(a,b)$.

(pf in App. F).

Clairaut's thm gives a condition where the mixed partials are equal.

<u>Note</u>: Clairaut's Thm was used heavily when solving exact equations in Math 230.