

## 14.3: Partial Derivatives.

14.3  
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The R.G. covered the basics.

Picture partial derivatives (Mathematica).

ex1 Consider  $f(x, y) = xy^2 + x^3$

(a) Find  $f_x(x, y) = y^2 + 3x^2$

We also write this as  $\frac{\partial F}{\partial x}$  or  $f_x$

(b) Find & interpret  $f_y(1, 2) = 4$

Note:  $x$  is fixed...  $y$  varies.

pts  $(1, y) \mapsto f(1, y) = y^2 + 1$

(see Mathematica)

ex2: Find  $\frac{\partial z}{\partial x}$  of  $\cos(xyz) = 3x + 2y + z$

implicit  
diff

$$\Rightarrow -\sin(xyz) \cdot (yz + xy \frac{\partial z}{\partial x}) = 3 + \frac{\partial z}{\partial x}$$

solve  
for  $\frac{\partial z}{\partial x}$

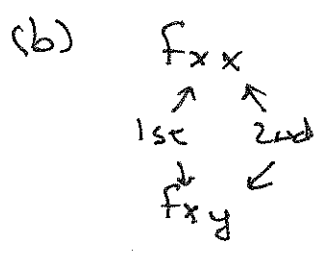
$$\Rightarrow -yz \sin(xyz) - xy \sin(xyz) \frac{\partial z}{\partial x} - 3 = \frac{\partial z}{\partial x}$$

$$\Rightarrow \frac{\partial z}{\partial x} = - \frac{yz \sin(xyz) + 3}{1 + xy \sin(xyz)}$$

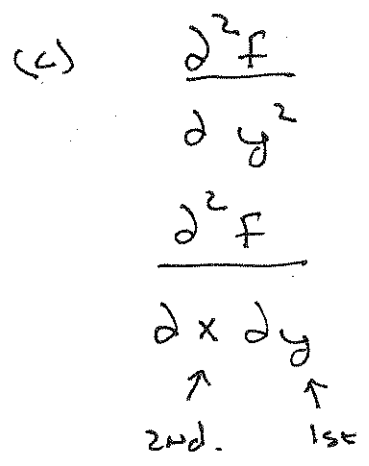
As w/ derivatives of fcts of 1 variable, we may be able to take higher order derivatives. However, we must choose which variable to differentiate wRT @ each step.

ex 3:  $f(x,y) = x^3 e^{5y} + y \sin(2x)$

(a)  $f_x$   
 $f_y$



This notation makes more sense when thought of as the differential operator.



for:  $f(x,y)$

1<sup>st</sup> d:  $\frac{\partial}{\partial x}(f(x,y)) = \frac{\partial f}{\partial x}$

2<sup>nd</sup> d:  $\frac{\partial}{\partial y} \left( \frac{\partial f}{\partial x} \right) = \frac{\partial^2 f}{\partial y \partial x}$

Notes: pure v. mixed partials.  
mixed partials are sometimes equal.

Clairaut's Thm: Suppose  $f$  is defined on a disk  $D$  that contains the pt  $(a,b)$ . If the fcts  $f_{xy}$  &  $f_{yx}$  are both cont on  $D$ , then  $f_{xy}(a,b) = f_{yx}(a,b)$ .

(pf in App. F).

Clairaut's Thm gives a condition where the mixed partials are equal.

Note: Clairaut's Thm was used heavily when solving exact equations in Math 230.