

14.1: Functions of Several Variables

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(I) Functions of two variables

$$f: (x, y) \longmapsto z$$

$$f: \mathbb{R}^2 \longmapsto \mathbb{R}$$

$$z = f(x, y)$$

Three notations.

 x, y : independent variables z : dependent variable.

ex: Tabular Tax Table example. (on page 4)

ex: consider $f(x, y) = \sqrt{y} + \sqrt{25 - x^2 - y^2}$

(a) find $f(1, 2)$

(b) find $f(-1, 0)$

(c) find $f(3, 6)$

(d) find/sketch the domain of f .

ex: Find/sketch the domain of $g(x, y) = \ln(16 - x^2 - y^2)$

(II) Graphs: If $z = f(x, y)$ is a fcn w/ domain \mathbb{D} , then

the graph of f is $\{(x, y, z) \in \mathbb{R}^3 \mid z = f(x, y) \text{ and } (x, y) \in \mathbb{D}\}$

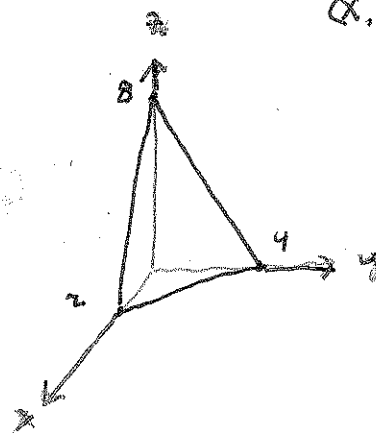
ex: sketch $f(x, y) = 8 - 4x - 2y$

plane w/ intercepts

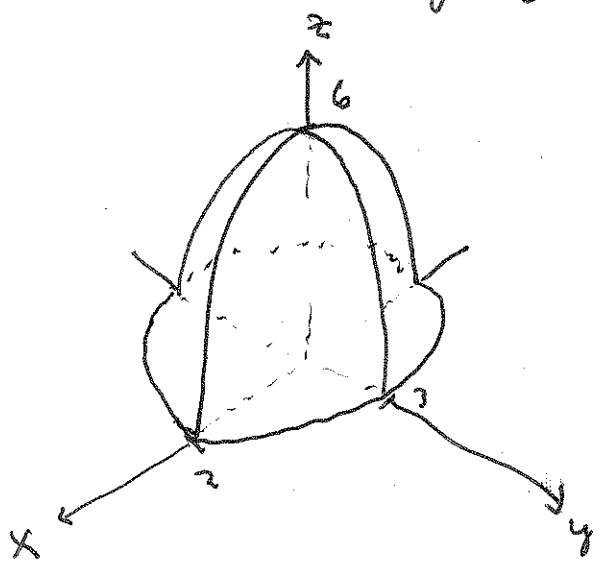
$$(0, 0, 8)$$

$$(0, 4, 0)$$

$$(2, 0, 0)$$



ex: sketch $g(x, y) = \sqrt{36 - 9x^2 - 4y^2}$



(III) LEVEL CURVES: The level curves of $z = f(x, y)$ are the curves $f(x, y) = k$ where k is a constant in the range of f .

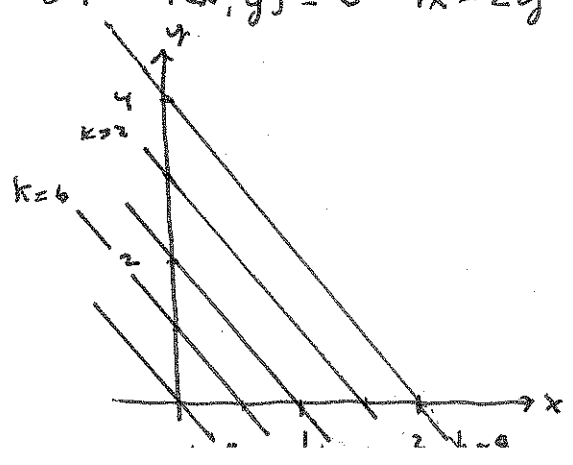
ex: Topo map (the geology dept. has lots of great examples)
or pages

ex: Isotherms. (the curves that separate colored bands)

A contour (or topographical) map is a graph containing equally spaced level curves.

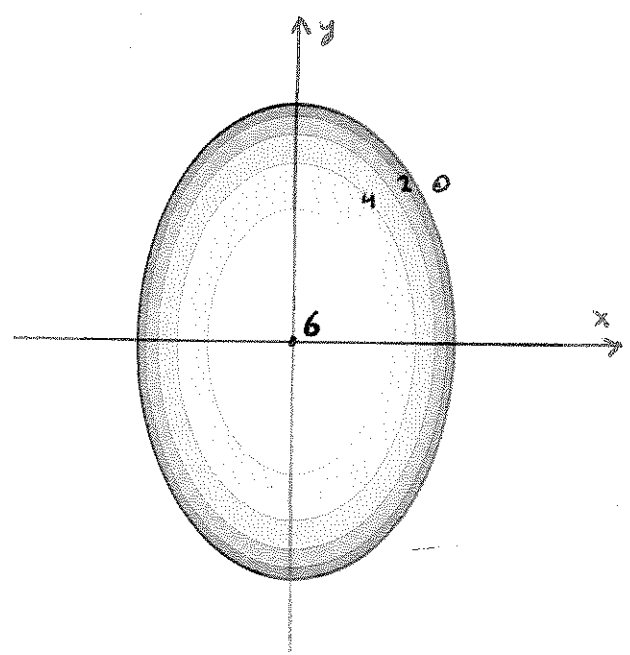
ex: sketch a contour map of $f(x, y) = 8 - 4x - 2y$

k	$k = f(x, y)$
$k = 0$	$4x + 2y = 8$
$k = 2$	$4x + 2y = 6$
$k = 4$	$4x + 2y = 4$
$k = 6$	$4x + 2y = 2$
$k = 8$	$4x + 2y = 0$



ex: sketch the level curves of $g(x, y) = \sqrt{36 - 9x^2 - 4y^2}$

k	$k = g(x, y)$
0	$0 = \sqrt{36 - 9x^2 - 4y^2}$ $= 36 - 9x^2 - 4y^2$ $\Rightarrow 36 = 9x^2 - 4y^2$
2	$2 = \sqrt{36 - 9x^2 - 4y^2}$ $\Rightarrow 32 = 9x^2 - 4y^2$
4	$4 = \sqrt{36 - 9x^2 - 4y^2}$ $\Rightarrow 20 = 9x^2 - 4y^2$
6	$6 = \sqrt{36 - 9x^2 - 4y^2}$ $\Rightarrow 0 = 9x^2 - 4y^2$



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Representations of Multivariate Functions

Tabular Data

The table below lists the 2004 tax owed for a married couple filing jointly with the given AGI and number of children. It assumes that the filers claim the standard deduction. It ignores all variables other than number of children. This includes the Alternative Minimum Tax and the Earned Income Credit.

Number of Children	0	1	2	3	4	5	6	7
AGI								
\$ -	\$ -	\$ (1,000.00)	\$ (2,000.00)	\$ (3,000.00)	\$ (4,000.00)	\$ (5,000.00)	\$ (6,000.00)	\$ (7,000.00)
\$ 5,000.00	\$ -	\$ (1,000.00)	\$ (2,000.00)	\$ (3,000.00)	\$ (4,000.00)	\$ (5,000.00)	\$ (6,000.00)	\$ (7,000.00)
\$ 10,000.00	\$ -	\$ (1,000.00)	\$ (2,000.00)	\$ (3,000.00)	\$ (4,000.00)	\$ (5,000.00)	\$ (6,000.00)	\$ (7,000.00)
\$ 15,000.00	\$ -	\$ (1,000.00)	\$ (2,000.00)	\$ (3,000.00)	\$ (4,000.00)	\$ (5,000.00)	\$ (6,000.00)	\$ (7,000.00)
\$ 20,000.00	\$ 410.00	\$ (900.00)	\$ (2,000.00)	\$ (3,000.00)	\$ (4,000.00)	\$ (5,000.00)	\$ (6,000.00)	\$ (7,000.00)
\$ 25,000.00	\$ 910.00	\$ (400.00)	\$ (1,710.00)	\$ (3,000.00)	\$ (4,000.00)	\$ (5,000.00)	\$ (6,000.00)	\$ (7,000.00)
\$ 30,000.00	\$ 1,410.00	\$ 100.00	\$ (1,210.00)	\$ (2,520.00)	\$ (3,830.00)	\$ (5,000.00)	\$ (6,000.00)	\$ (7,000.00)
\$ 35,000.00	\$ 2,150.00	\$ 685.00	\$ (710.00)	\$ (2,020.00)	\$ (3,330.00)	\$ (4,640.00)	\$ (5,950.00)	\$ (7,000.00)
\$ 40,000.00	\$ 2,900.00	\$ 1,435.00	\$ (30.00)	\$ (1,495.00)	\$ (2,830.00)	\$ (4,140.00)	\$ (5,450.00)	\$ (6,760.00)
\$ 45,000.00	\$ 3,650.00	\$ 2,185.00	\$ 720.00	\$ (745.00)	\$ (2,210.00)	\$ (3,640.00)	\$ (4,950.00)	\$ (6,260.00)
\$ 50,000.00	\$ 4,400.00	\$ 2,935.00	\$ 1,470.00	\$ 5.00	\$ (1,460.00)	\$ (2,925.00)	\$ (4,390.00)	\$ (5,760.00)
\$ 55,000.00	\$ 5,150.00	\$ 3,685.00	\$ 2,220.00	\$ 755.00	\$ (710.00)	\$ (2,175.00)	\$ (3,640.00)	\$ (5,105.00)
\$ 60,000.00	\$ 5,900.00	\$ 4,435.00	\$ 2,970.00	\$ 1,505.00	\$ 40.00	\$ (1,425.00)	\$ (2,890.00)	\$ (4,355.00)
\$ 65,000.00	\$ 6,650.00	\$ 5,185.00	\$ 3,720.00	\$ 2,255.00	\$ 790.00	\$ (675.00)	\$ (2,140.00)	\$ (3,605.00)
\$ 70,000.00	\$ 7,400.00	\$ 5,935.00	\$ 4,470.00	\$ 3,005.00	\$ 1,540.00	\$ 75.00	\$ (1,390.00)	\$ (2,855.00)
\$ 75,000.00	\$ 8,250.00	\$ 6,685.00	\$ 5,220.00	\$ 3,755.00	\$ 2,290.00	\$ 825.00	\$ (640.00)	\$ (2,105.00)
\$ 80,000.00	\$ 9,500.00	\$ 7,725.00	\$ 5,970.00	\$ 4,505.00	\$ 3,040.00	\$ 1,575.00	\$ 110.00	\$ (1,355.00)
\$ 85,000.00	\$ 10,750.00	\$ 8,975.00	\$ 7,200.00	\$ 5,425.00	\$ 3,790.00	\$ 2,325.00	\$ 860.00	\$ (605.00)
\$ 90,000.00	\$ 12,000.00	\$ 10,225.00	\$ 8,450.00	\$ 6,675.00	\$ 4,900.00	\$ 3,125.00	\$ 1,610.00	\$ 145.00
\$ 95,000.00	\$ 13,250.00	\$ 11,475.00	\$ 9,700.00	\$ 7,925.00	\$ 6,150.00	\$ 4,375.00	\$ 2,600.00	\$ 895.00
\$ 100,000.00	\$ 14,500.00	\$ 12,725.00	\$ 10,950.00	\$ 9,175.00	\$ 7,400.00	\$ 5,625.00	\$ 3,850.00	\$ 2,075.00

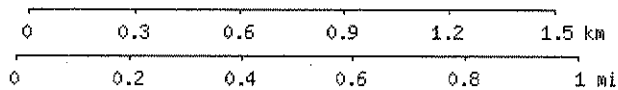
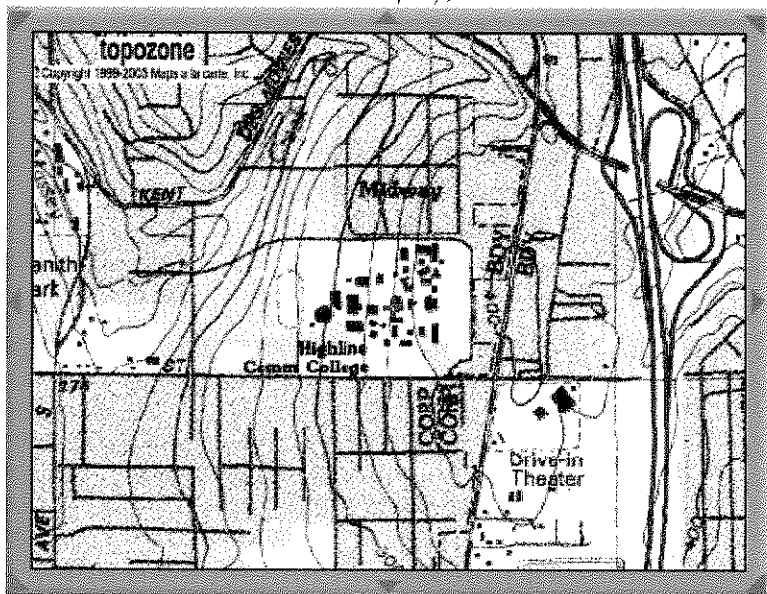
The Topographical Map

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UTM 10 552800E 5248568N (WGS84/NAD83)

USGS Des Moines Quad

View *TopoZone Pro* aerial photos, shaded relief, street maps, interactive coordinate display, and elevation data

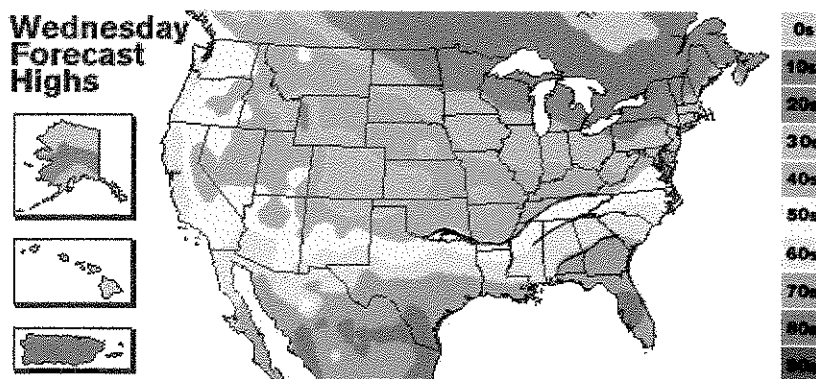


This topographical map is courtesy of Topozone.com (a second example is Longmire).

Algebraic Contour Plots

A webMathematica Contour Plotter, was formerly found [here](#). Note that the lighter the shading, the higher the point.

The Weather Map (Showing isotherms).



iso therms = temperature level curves

This weather map (from 02/23/05) is courtesy of the USA Today.

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