Test 2
Dusty Wilson
Math 153

No work = no credit

No Symbolic Calculators

Name: Key

Notable enough, however, are the controversies over the series 1 - 1 + 1 - 1 + 1 - ... whose sum was given by Leibniz as 1/2, although others disagree. ... Understanding of this question is to be sought in the word "sum"; this idea, if thus conceived -- namely, the sum of a series is said to be that quantity to which it is brought closer as more terms of the series are taken -- has relevance only for convergent series, and we should in general give up the idea of sum for divergent series.

Leonard Euler (1707 - 1783)
Swiss mathematician

$\frac{\frac{5}{3}}{1-\frac{1}{3}}$

Warm-ups (1 pt each): $\sum_{n=1}^{\infty} 5\cdot\left(\frac{1}{3}\right)^n = \underline{\frac{5}{2}}$ $\quad \sum_{n=1}^{\infty} \frac{1}{n} = \underline{\infty}$ $\quad \lim_{n\to\infty} \frac{3-7n^2}{3n^2+1} = \underline{-\frac{7}{3}}$

1.) (1 pt) According to Euler, for what type of infinite series is it relevant to calculate the sum? Answer using complete English sentences.

Sums only make sense for convergent series.

2.) (10 pts) Does $\sum_{n=1}^{\infty} \frac{5}{n\sqrt{n^2+1}}$ converge or diverge? Justify your answer.

$$\sum_{n=1}^{\infty} \frac{5}{n\sqrt{n^2+1}} < 5\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2}}$$

$$= 5 \underbrace{\sum_{n=1}^{\infty} \frac{1}{n^2}}_{\text{convergent } p\text{-Series.}}$$

Hence, the series converges by the comparison test.

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3.) (10 pts) Does $\sum_{n=1}^{\infty} \frac{(-1)^n 3^n (n+1)}{n!}$ diverge? If not, is it conditionally or absolutely convergent? Justify your answer.

Ratio Test

$$\lim_{n\to\infty} \left| \frac{\frac{(-1)^{n+1} 3^{n+1} (n+2)}{(n+1)!}}{\frac{(-1)^n 3^n (n+1)}{n!}} \right| = \lim_{n\to\infty} \frac{3(n+2)}{(n+1)^2} = 0 < 1$$

Hence the series is absolutely convergent by the ratio test.

4.) (10 pts) Does $\sum_{n=0}^{\infty} \frac{(-1)^n}{1+\sqrt{n}}$ converge or diverge? If it converges, is it conditional or absolute convergence? Justify your answer.

AST

$$\lim_{n\to\infty} \frac{1}{1+\sqrt{n}} = 0 \quad \text{so} \quad \sum \frac{(-1)^n}{1+\sqrt{n}} \quad \text{by the AST}$$

LCT

$$\lim_{n\to\infty} \frac{\frac{1}{1+\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{\sqrt{n}}{1+\sqrt{n}} = 1$$

since $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ is a divergent p-series, $\sum_{n=1}^{\infty} \frac{1}{1+\sqrt{n}} = \sum_{n=1}^{\infty} \left| \frac{(-1)^n}{1+\sqrt{n}} \right|$ diverges by the LCT. Hence $\sum \frac{(-1)^n}{1+\sqrt{n}}$ is conditionally convergent.

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5.) (10 pts) Determine whether the sequence $a_n = \left(1+\frac{5}{n}\right)^{3n}$ converges or diverges. If it converges, find the limit.

$$\lim_{n\to\infty}\left(1+\frac{5}{n}\right)^{3n} = e^{\lim_{n\to\infty} \ln\left[\left(1+\frac{5}{n}\right)^{3n}\right]}$$

$$= e^{\lim_{n\to\infty} 3n \ln\left(1+\frac{5}{n}\right)}$$

$$= e^{\lim_{n\to\infty} \frac{\ln\left(1+\frac{5}{n}\right)}{\frac{1}{3n}}}$$

(H) $$= e^{\lim_{n\to\infty} \frac{\frac{1}{1+\frac{5}{n}}\cdot -\frac{5}{n^2}}{-\frac{1}{3n^2}}}$$

$$= e^{\lim_{n\to\infty} \frac{15}{1+\frac{5}{n}}}$$

$$= e^{15}$$

6.) (10 pts) Write $\vec{a}(3)$ in the form $\vec{a} = a_T\vec{T} + a_N\vec{N}$ without finding $\vec{T}$ and $\vec{N}$ for the position vector-valued function $\vec{r}(t) = t^2\vec{i} + \left(t+\frac{t^3}{3}\right)\vec{j} + \left(t-\frac{t^3}{3}\right)\vec{k}$ at $t=3$. That is, you need to find a_T and a_N.

$$\vec{r}\,'(t) = \langle 2t, 1+t^2, 1-t^2\rangle\Big|_{t=3} \langle 6, 10, -8\rangle$$

$$\vec{r}\,''(t) = \langle 2, 2t, -2t\rangle\Big|_{t=3} \langle 2, 6, -6\rangle$$

$$a_T = \frac{\vec{r}\,'\cdot\vec{r}\,''}{|\vec{r}\,'|}$$

$$= \frac{120}{\sqrt{200}}$$

$$a_N = \frac{|\langle -12, 20, 16\rangle|}{\sqrt{200}}$$

$$= \frac{\sqrt{800}}{\sqrt{200}}$$

$$= 2$$

ur. 4

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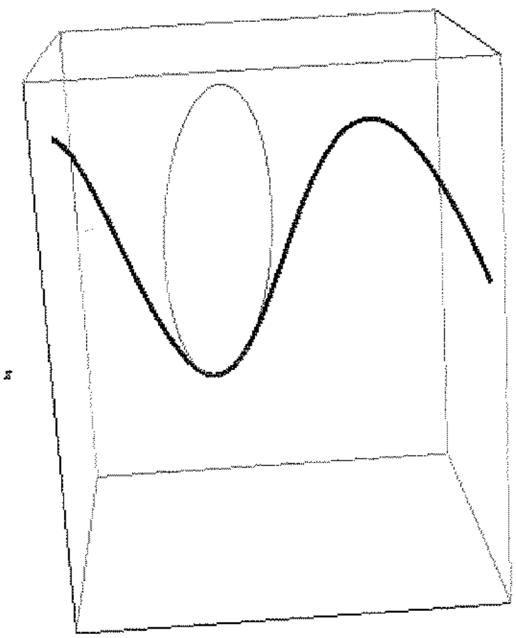
17.) (10 pts) Find the kissing circle of $\vec{r}(t) = \langle 6\sin(2t), 5t, 6\cos(2t)\rangle$ when $t = \pi/2$ given that at this t value:

$$\vec{T}(t) = \frac{1}{13}\langle 12\cos(2t), 5, -12\sin(2t)\rangle\Big|_{t=\pi/2} \frac{1}{13}\langle 12, 5, 0\rangle$$
$$\vec{N}(t) = \langle -\sin(2t), 0, -\cos(2t)\rangle\Big|_{t=\pi/2} \langle 0, 0, -1\rangle$$
$$\vec{r}\,'(t) = \langle 12\cos(2t), 5, -12\sin(2t)\rangle\Big|_{t=\pi/2} \langle 12, 5, 0\rangle$$
$$\vec{r}\,''(t) = \langle -24\sin(2t), 0, \cos(2t)\rangle\Big|_{t=\pi/2} \langle 0, 0, -24\rangle$$



$$K = \frac{|\vec{r}\,' \times \vec{r}\,''|}{|\vec{r}\,'|^3}$$
$$= \frac{|\langle -120, 288, 0\rangle|}{13^3}$$
$$= \frac{312}{2197}$$
$$= \frac{24}{169}$$

$$\text{kiss}(\theta) = \langle 0, \tfrac{5\pi}{2}, -6\rangle + \frac{169}{24}\langle 0,0,-1\rangle + \frac{169}{24}\left(\tfrac{1}{13}\langle 12,5,0\rangle\cos\theta + \langle 0,0,-1\rangle\sin\theta\right)$$

point

shift

circle.

18.) (10 pts) Find the arclength of $\vec{r}(t) = \langle e^t \cos(t), e^t \sin t, e^t \rangle$ on $-\ln 4 \le t \le 0$. Hint: You must use the product rule.

$$\vec{r}\,'(t) = \langle e^t \cos t - e^t \sin t,\ e^t \sin t + e^t \cos t,\ e^t \rangle$$

$$\Rightarrow |\vec{r}\,'(t)| = \sqrt{(e^{2t}\cos^2 t - 2e^{2t}\cancel{\sin t \cos t} + e^{2t}\sin^2 t + e^{2t}\sin^2 t + 2e^{2t}\cancel{\sin t \cos t} + e^{2t}\cos^2 t + e^{2t})}$$

$$= e^t\sqrt{1+1+1}$$

$$\Rightarrow \text{Arclength} = \int_{-\ln 4}^{0} e^t \sqrt{3}\, dt$$

$$= \left[e^t \sqrt{3} \right]_{-\ln 4}^{0}$$

$$= \sqrt{3}\left(1 - \frac{1}{4}\right)$$

$$= \frac{3\sqrt{3}}{4}$$

Test #2

	7-10	0 - 6
2	卌 ||||	卌 卌 ||
3	卌 卌 卌 |	卌
4	卌 |||	卌 卌 |||
5	卌 卌 ~~卌~~ ||||	卌 ||
6	卌 卌 卌 ||	卌
7	卌 ||||	卌 卌 ||
8	卌 卌 |||	卌 |||

100	90's	80's	70's	60's	< 60
0	3	4	4	1	10