high 100%

$\bar{x}$ 79.75%

med 82.9%

Test 1 (Version φ)
Dusty Wilson
Math 153

Name: key

No work = no credit

No Symbolic Calculators

I myself, a professional mathematician, on re-reading my own work find it strains my mental powers to recall to mind from the figures the meanings of the demonstrations, meanings which I myself originally put into the figures and the text from my mind. But when I attempt to remedy the obscurity of the material by putting in extra words, I see myself falling into the opposite fault of becoming chatty in something mathematical.

Johannes Kepler (1597 - 1630)
German astronomer

Warm-ups (1 pt each): $\vec{i} \times \vec{i} = \vec{0}$ $\quad \vec{j} \cdot \vec{k} = 0$ $\quad \vec{k} - \vec{i} = \langle -1, 0, 1 \rangle$

1.) (1 pt) Based upon the quote above, how easily did Kepler understand his earlier work? Answer using complete English sentences.

2.) ($\cancel{12}$ 10 pts) Consider $\vec{u} = \langle 3, 6, 9 \rangle$ and $\vec{v} = \langle 2, 7, 1 \rangle$

a.) Find $\vec{u} - 2\vec{v}$

$\langle -1, -8, 7 \rangle$

b.) Find $\vec{u} \cdot \vec{v}$

$6 + 42 + 9 = 57$

c.) Find a vector parallel to $\vec{v}$ w/ length 3.

$$\frac{3\vec{v}}{|\vec{v}|} = \frac{3}{\sqrt{54}} \langle 2, 7, 1 \rangle$$

d.) Find the angle between $\vec{u}$ & $\vec{v}$. (rad to 2 decimal places)

$$\theta = \cos^{-1}\left(\frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}\right)$$

$$= \cos^{-1}\left(\frac{57}{\sqrt{126}\sqrt{54}}\right)$$

$$\approx 0.81 \text{ OR } 46.3^\circ$$

(e.) find $\vec{u} \times \vec{v}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 6 & 9 \\ 2 & 7 & 1 \end{vmatrix} = \langle -57, 15, 9 \rangle$$

3.) (~~12~~ 10 pts) Consider the plane $4x+3y+2z=1$

(6 pts)

a.) Find the equation of the line that is normal to the plane through point $A(1,6,2)$. Give your answer parametrically.

$$\vec{r}(t) = \langle 1, 6, 2 \rangle + t \langle 4, 3, 2 \rangle$$

$$= \langle 1+4t,\ 6+3t,\ 2+2t \rangle$$

(4 pts)

b.) Find the distance from the plane to the point $A(1,5,9)$.

Formula: Distance from (x_1, y_1, z_1) to $ax+by+cz+d=0$.

is $D = \dfrac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2+b^2+c^2}}$

$$D = \frac{|4(1)+3(5)+2(9)-1|}{\sqrt{16+9+4}}$$

$$= \frac{36}{\sqrt{29}}$$

4.) (10 pts) Find an equation of the plane that passes through the point $(-1, 1, 3)$ and contains the vectors $\vec{u} = \langle 4,-1,5 \rangle$ and $\vec{v} = \langle -9,2,6 \rangle$.

Find a normal vector.

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -1 & 5 \\ -9 & 2 & 6 \end{vmatrix}$$

$$= \langle -16, -69, -1 \rangle$$

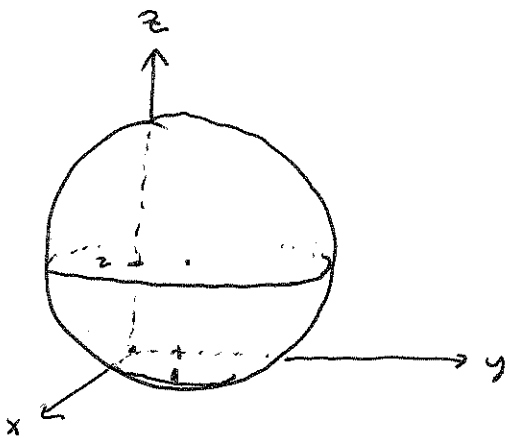
plane: $-16(x+1) - 69(y-1) - 1(z-3) = 0$

$$\Rightarrow -16x - 69y - z = -55$$

OR $16x + 69y + z = 55$

5.) (5 pts) What is the equation of the sphere with radius 3, centered at the point (0,1,2)? Sketch the sphere.

$$x^2 + (y-1)^2 + (z-2)^2 = 9$$



6.) (10 pts) Find the equation of the tangent line to the curve parameterized by $x = 4\cos(t)$ and $y = t^2$ when $t = \frac{\pi}{6}$

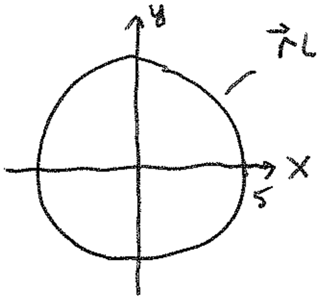
slope: $\frac{dy}{dx} = \frac{2t}{-4\sin t}\Big|_{t=\frac{\pi}{6}} \quad \frac{\frac{\pi}{3}}{-4(\frac{1}{2})} = -\frac{\pi}{6}$

point: $\langle 4\cos(t), t^2\rangle\Big|_{t=\frac{\pi}{6}} \quad \langle 4\cdot\frac{\sqrt{3}}{2}, \frac{\pi^2}{36}\rangle = \langle 2\sqrt{3}, \frac{\pi^2}{36}\rangle$

Tangent line: $y - \frac{\pi^2}{36} = -\frac{\pi}{6}(x - 2\sqrt{3})$

7.) (10 pts) Use techniques developed in this course to verify that the circumference of a circle with radius 5 is 10π.

Hint: Begin by writing a parametric equation for a circle of radius 5 centered at the origin.



$\vec{r}(t) = \langle 5\cos(t), 5\sin(t) \rangle$

$$\text{circumference} = 4\int_0^{\pi/2} \sqrt{(-5\sin t)^2 + (5\cos t)^2}\, dt$$
$$= 4\int_0^{\pi/2} 5\, dt$$
$$= 4 \cdot 5 \cdot \frac{\pi}{2}$$
$$= 10\pi$$

8.) (10 pts) Set up an integral to find the area inside the circle $r = 2$, but outside the cardiod $r = 4(1-\cos\theta)$.

Note: You may evaluate the integral to verify the area is $14\sqrt{3} - \frac{20\pi}{3}$

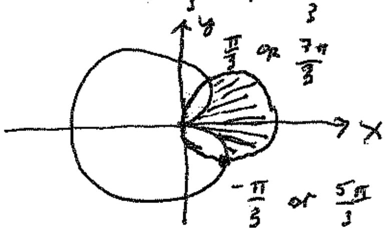
Find the intersection points.

$2 = 4(1-\cos\theta)$

$\Rightarrow \frac{1}{2} = 1 - \cos\theta$

$\Rightarrow \cos\theta = \frac{1}{2}$

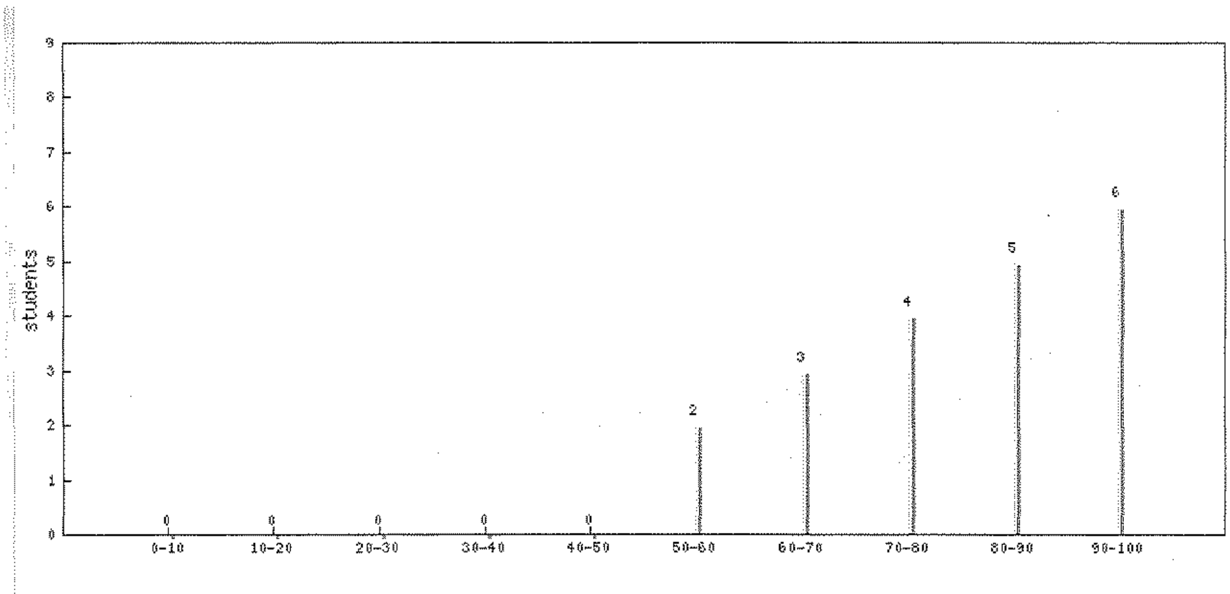
$\Rightarrow \theta = \frac{\pi}{3}$ or $\frac{5\pi}{3}$



$$\text{Area} = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{2}\cdot 2^2\, d\theta - \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{2}\left[4(1-\cos\theta)\right]^2 d\theta$$
$$= 2\cdot\frac{1}{2}\int_0^{\pi/3} \left[4 - 16(1-\cos\theta)^2\right] d\theta$$
$$= \int_0^{\pi/3} (-12 + 32\cos\theta - 16\underbrace{\cos^2\theta}_{\frac{1+\cos 2\theta}{2}})\, d\theta$$
$$= \left[-12\theta + 32\sin\theta - 8\theta - 4\sin 2\theta\right]_0^{\pi/3}$$
$$= -\frac{20\pi}{3} + \frac{32\sqrt{3}}{2} - \frac{4\sqrt{3}}{2}$$
$$= 14\sqrt{3} - \frac{20\pi}{3}$$

Course Objectives: *The student will be able to ...*

1. Use derivatives to graph parametrically defined curves
2. Find areas and arc lengths for polar and parametric graphs
3. Apply properties of vectors, including dot and cross products
4. Graph elementary equations in three dimensions, find equations of lines and planes, and use vector properties to calculate distances and relationships for lines, points and planes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