Group Quiz 4
Dusty Wilson
Math 153

Name: Key

No work = no credit

1.) Use a convergence test (sections 11.3-5) to determine whether the series $\sum_{n=7}^{\infty} \frac{1}{n^2-5n-6}$ converges or diverges. If it converges, find the exact sum. Justify your answer.

LCT $\lim_{n\to\infty} \frac{\frac{1}{n^2-5n-6}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^2}{n^2-5n-6} = 1$

Hence $\sum_{n=7}^{\infty} \frac{1}{n^2-5n-6}$ converges by the LCT since $\sum_{n=7}^{\infty} \frac{1}{n^2}$ is a convergent p-series.

Telescoping

$\sum_{n=7}^{\infty} \frac{1}{n^2-5n-6} = \sum_{n=7}^{\infty} \left(\frac{+1/7}{n-6} + \frac{-1/7}{n+1} \right) = \frac{1}{7}\left[\left(\frac{1}{1} + \cancel{\frac{-1}{8}}\right) + \left(\frac{1}{2} + \cancel{\frac{-1}{9}}\right) + \left(\frac{1}{3} - \cancel{\frac{1}{10}}\right) + \left(\frac{1}{4} - \cancel{\frac{1}{11}}\right) + \left(\frac{1}{5} - \cancel{\frac{1}{12}}\right) + \left(\frac{1}{6} - \cancel{\frac{1}{13}}\right) + \left(\frac{1}{7} - \cancel{\frac{1}{14}}\right) + \left(\cancel{\frac{1}{8}} - \cancel{\frac{1}{15}}\right) + \cdots \right]$

which simplifies to

$= \frac{1}{7}\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{7}\right) = \boxed{\frac{363}{980}}$

This is the exact sum

2.) Determine whether the sequence $a_n = \left(1 - \frac{2}{3n}\right)^{5n}$ converges or diverges. If it converges, find the limit.

$\lim_{n\to\infty} \left(1 - \frac{2}{3n}\right)^{5n} = \lim_{x\to\infty} \left(1 + \frac{-2}{3x}\right)^{5x}$

$= \lim_{x\to\infty} e^{\ln\left[\left(1 + \frac{-2}{3x}\right)^{5x}\right]}$

$= \lim_{x\to\infty} e^{5x \ln\left(1 - \frac{2}{3x}\right)}$

$= e^{\lim_{x\to\infty} \frac{\ln\left(1 - \frac{2}{3x}\right)}{\frac{1}{5x}}}$

$\overset{H}{=} e^{\lim_{x\to\infty} \frac{\frac{1}{1 - \frac{2}{3x}} \cdot \frac{2}{3x^2}}{-\frac{1}{5x^2}}}$

$= e^{\lim_{x\to\infty} \frac{3x}{3x-2} \cdot \frac{-10}{3}}$

$= e^{-\frac{10}{3}}$

3.) Test the series $\sum_{n=1}^{\infty} \frac{3}{n\sqrt{\ln n} + 4}$ for convergence or divergence? Justify your answer.

$$\sum_{n=1}^{\infty} \frac{3}{n\sqrt{\ln n} - 4} = -\frac{3}{4} + \sum_{n=2}^{\infty} \frac{3}{n\sqrt{\ln n} - 4}$$

$$\geq -\frac{3}{4} + \sum_{n=2}^{\infty} \frac{3}{n\sqrt{\ln n}}$$

$$= \infty$$

so the series diverges by the comparison test.

Integral Test

$$\int_2^{\infty} \frac{3}{x\sqrt{\ln x}}\,dx = \lim_{t\to\infty} \int_2^{t} \frac{3}{x\sqrt{\ln x}}\,dx$$

let $u = \ln x$

$du = \frac{1}{x}dx$

$$= \lim_{t\to\infty} \int_{\ln 2}^{\ln t} \frac{3}{\sqrt{u}}\,du$$

$$= \lim_{t\to\infty} \Big[6\sqrt{u} \Big]_{\ln 2}^{\ln t}$$

$$= \lim_{t\to\infty} \left(6\sqrt{\ln t} - 6\sqrt{\ln 2} \right)$$

$$= \infty$$

Hence $\sum_{n=2}^{\infty} \frac{3}{n\sqrt{\ln n}}$ diverges by the integral test